Application of an Adjoint SWAN Wave Model

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Outline

- Introduction
- SWAN Adjoint Overview
- Case Studies
  - Santa Rosa Island, Florida
  - NCEX
  - Duck, NC
- Summary and Conclusions
- Future Plans

Motivation

Forecasts of wave and current conditions used in mission planning, AUV & UUV guidance, Mine Warfare

- Under-estimate: Lives and equipment endangered
- Over-estimate: Loss of time and money

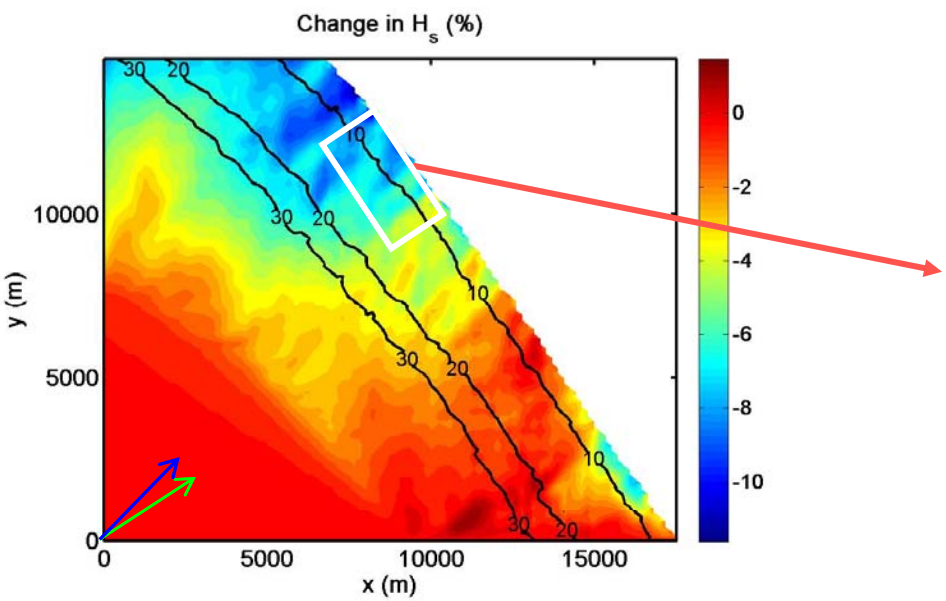
USN is investing in number of different types of sensors

- Use data for more than model verification?
- Need guidance in deployment: How many & where?

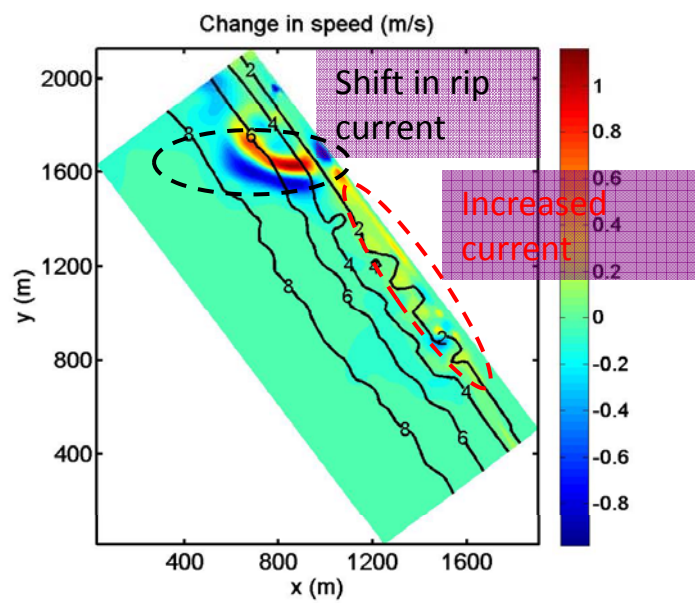
The Problem

- Errors in wave condition at boundary cause errors in wave heights (H) closer to shore
- Currents in the near-shore are driven by gradients in wave stress (proportional to H^2) => very sensitive to the wave conditions

% Change in wave height due to 5° change in incident wave angle



Corresponding change in current (from Delft3D)



Littoral Sensing and Modeling

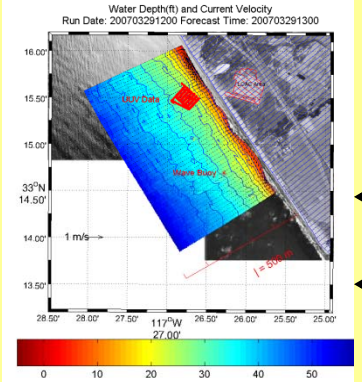
Forward unit sensor deployment

Where should sensors be deployed?

Sensor Placement Toolkit

Remotely sensed data

Comms /data flow



Model prediction with data assimilation

Navy Center

Tools for model setup and evaluation

Exercises to test CONOPS

Wave Data Assimilation: Walker (2001)

Model equation (SWAN):

$$\frac{\partial N}{\partial t} + \frac{\partial c_x N}{\partial x} + \frac{\partial c_y N}{\partial y} + \frac{\partial c_\sigma N}{\partial \sigma} + \frac{\partial c_\theta N}{\partial \theta} = \frac{S_{tot}}{\sigma}$$

N	: Wave action (E/σ)
$c_x, c_y, c_\sigma, c_\theta$	: Propagation speed
S_{tot}	: Source/Sink terms

$$S_{tot} = S_{ds,b} + S_{ds,w} + S_{ds,br} + S_{in} + S_{nl}$$

$S_{ds,b}$	: Bottom friction (sink)
$S_{ds,w}$	: White capping (sink)
$S_{ds,br}$	: Depth-limited breaking (sink)
S_{in}	: Wind input (source)
S_{nl}	: Non-linear interactions

Underlying assumptions for adjoint model:

Fetch limited sea state

Model itself is perfect (i.e. all errors propagate in from the boundary)

Errors at the offshore boundary are uniformly distributed along the boundary

Data used to correct the boundary conditions are from observations *outside* the surf zone

Assimilation methodology

Objective function:

$$J = \int_{\mathcal{R}} \int_{\mathcal{S}} \left\{ \frac{1}{M} \sum_{i=1}^{M} (E - \widehat{E}_i)^2 \delta(\mathbf{x} - \mathbf{x}_i) + A \, \widetilde{\nabla} \cdot (\widetilde{\mathbf{C}} N) \right\} ds \, d\mathbf{x} + \phi \int_{\mathcal{S}} E_b^2 \, ds$$

M	: Number of observation locations
E(**x,s**)	: Energy spectrum
A(**x,s**)	: Lagrange multiplier - adjoint wave action spectrum
E_b	: Boundary spectrum

Penalty on the control variable (second integral) ensures unique solution (Bennet and Miller, 1991)

Veeramony, J, D. Walker and L. Hsu, (2010), "A variational data assimilation system for nearshore applications of SWAN", Ocean Modeling, 35, 206-214.
Walker, D., (2006): "Assimilation of SAR imagery in a nearshore spectral wave model.", Tech Report 200236, GDAIS.

Assimilation methodology

Cost function (diagnostic):

$$J = \frac{1}{M}\sum_{i=1}^{M}\int_{\mathcal{S}}\left[E(\mathbf{x}_i,\mathrm{s}) - \widehat{E}_i(\mathrm{s})\right]^2 d\mathrm{s} + \phi\int_{\mathcal{S}} E_b(\mathrm{s})^2\, d\mathrm{s}$$

used to determine convergence

Forward model
(Stationary SWAN):

$$\widetilde{\nabla}\cdot(\widetilde{\mathrm{C}}N) = \frac{S_{tot}}{\sigma}$$

Adjoint SWAN model:

$$\widetilde{\mathrm{C}}\,'\cdot\widetilde{\nabla}A = -\frac{2\sigma}{M}\sum_{i=1}^{M}(E-\widehat{E}_i)\delta(\mathbf{x}-\mathbf{x}_i)$$

A = adjoint spectrum at the boundary

Gradient of *J* (used to iteratively determine corrected boundary spectrum which minimizes J):

$$\frac{\partial J}{\partial E_b} = \int_{\partial\mathcal{R}}\frac{A}{\sigma}\,\widetilde{\mathrm{C}}\cdot\widehat{n}\,d\xi + 2\phi E_b$$

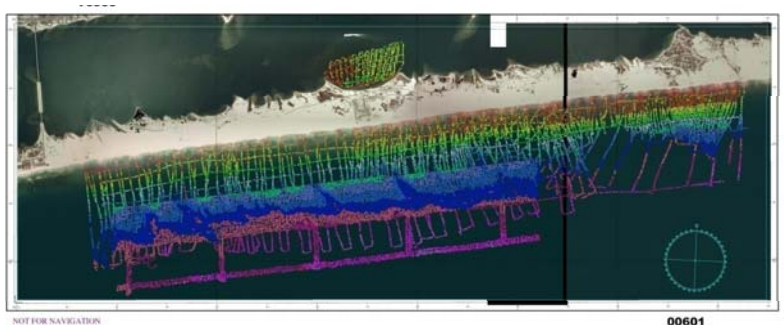
Experimental Resources (Eglin) Jan '09

Equipment:

<u>Over the Beach Operations</u>

- Raven B UAS

- Real Time Kinematic "Rover" GPS



Santa Rosa Island, FL

<u>Over Ocean Operations</u>

- Jetski (SBES)

- Safeboat Defender (MBES)

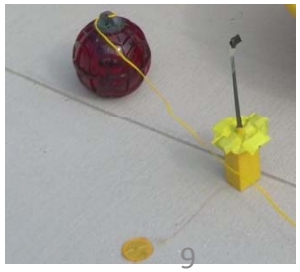
. Sea Fox

- Expeditionary Multibeam Kit (EMK)

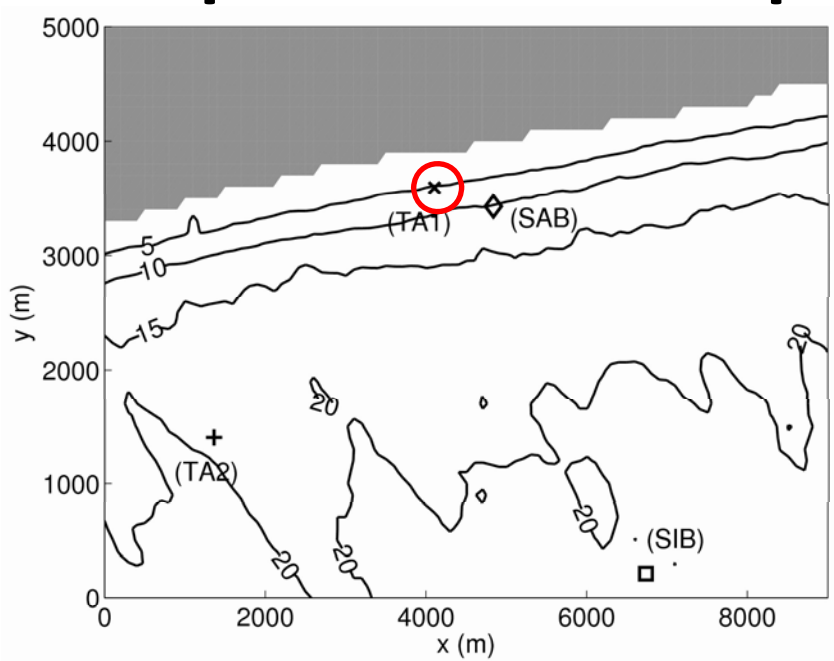
- PSI ADCP / Wave Buoy

Tri-AXYS Buoy

Drifting Buoys



Experimental Setup



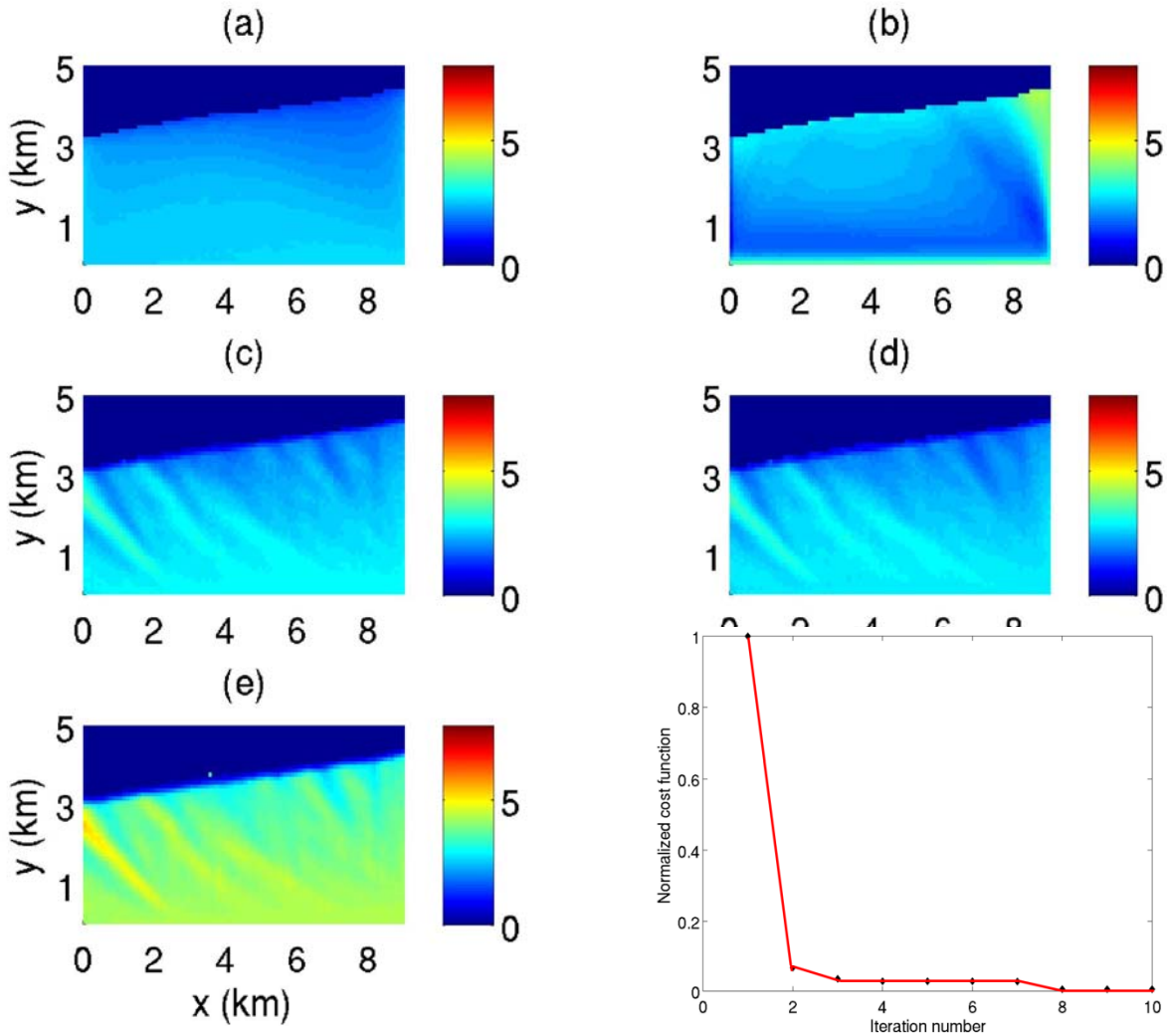
Wave spectra from TA1 used as input to assimilation system

Case	U_{10} (m/s)	H_{sig} (m)	T_m (s)
No winds In interior [a]	5	0.56	2.74
[c]	10	2.28	5.76
[e]	12	3.3	6.98
Cases b, d, f same as above, but with U_{10} acting in domain as well*			

Incident wave spectrum based on Pierson-Moskowitz spectrum

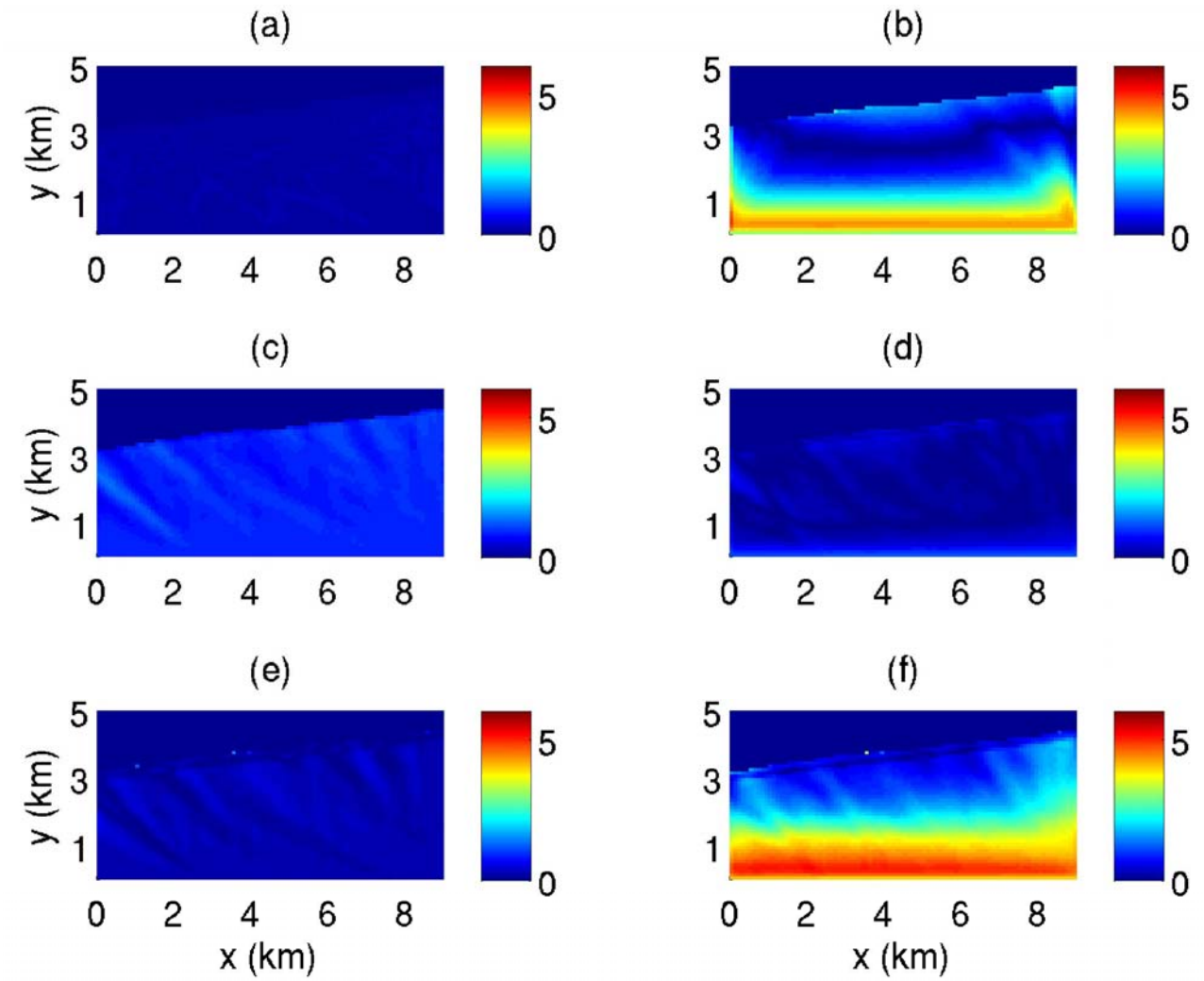
*Quadruplet interaction and dissipation due to white capping enabled in forward model

Difference between actual and modeled significant wave height (twin-experiments)



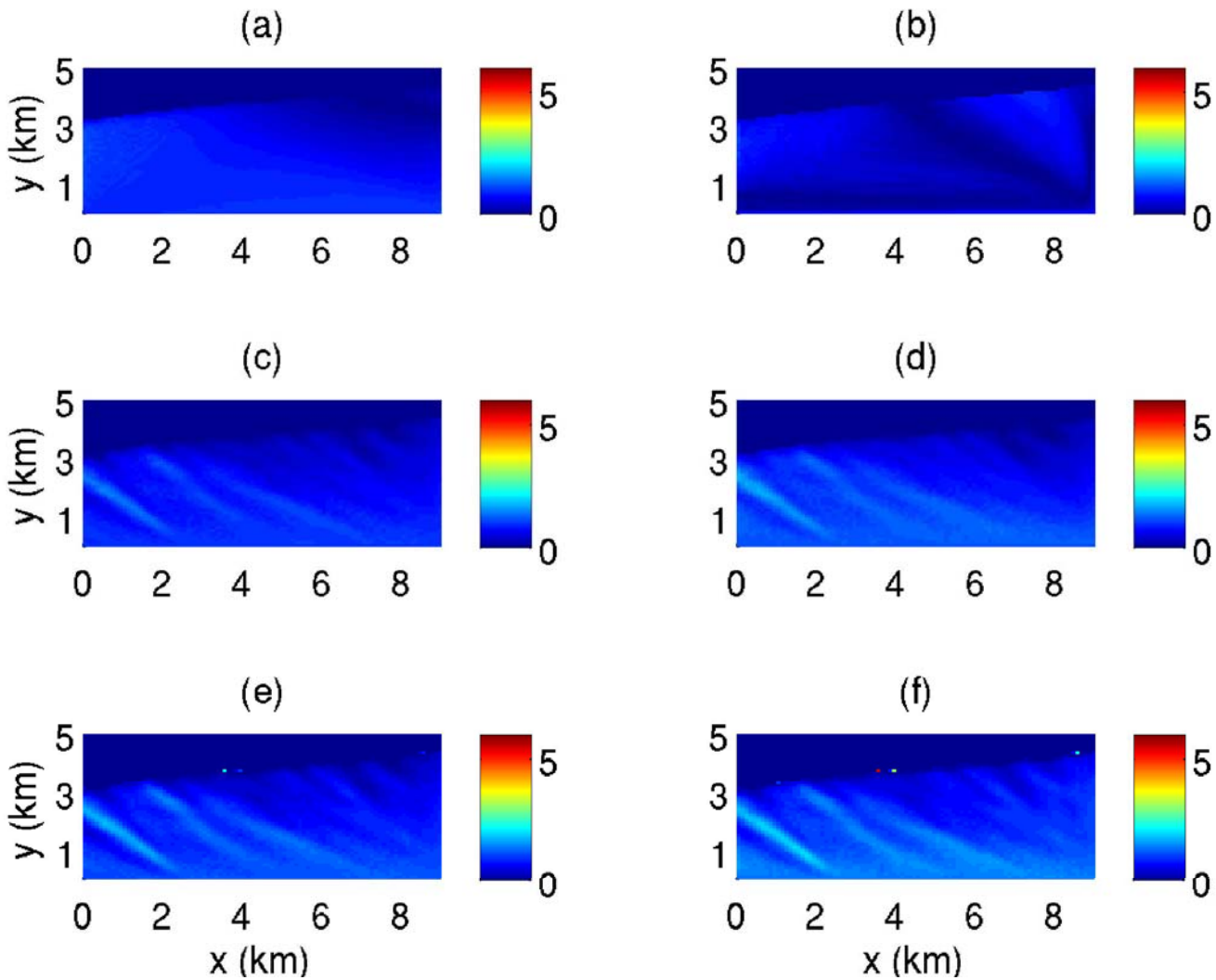
Shown as a percentage of the actual wave height for all six cases

Difference between actual and modeled mean wave period



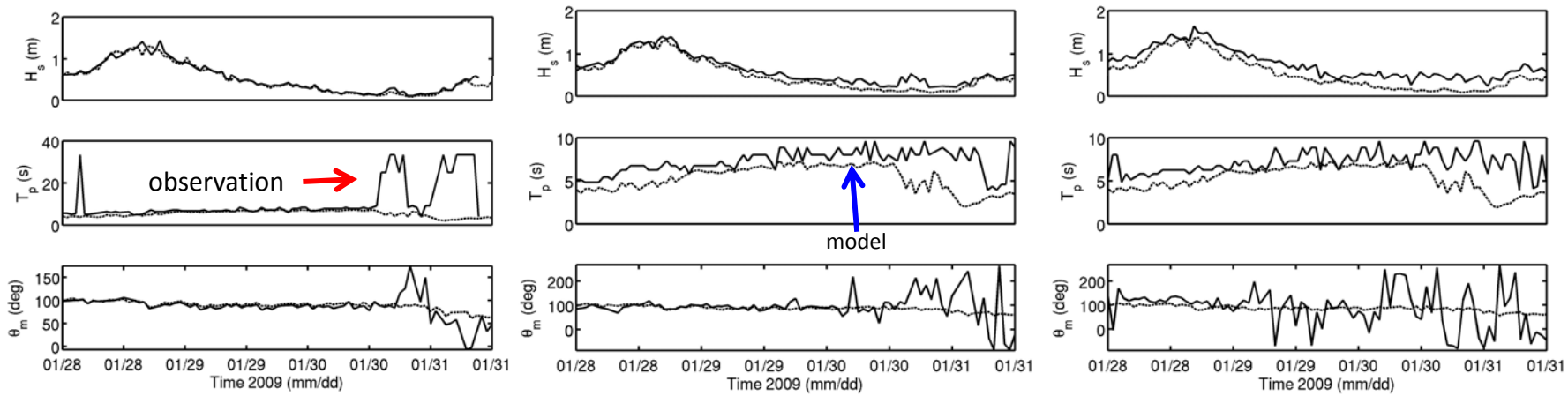
Shown as a percentage of the actual wave period for all six cases

Difference between actual and modeled mean wave direction

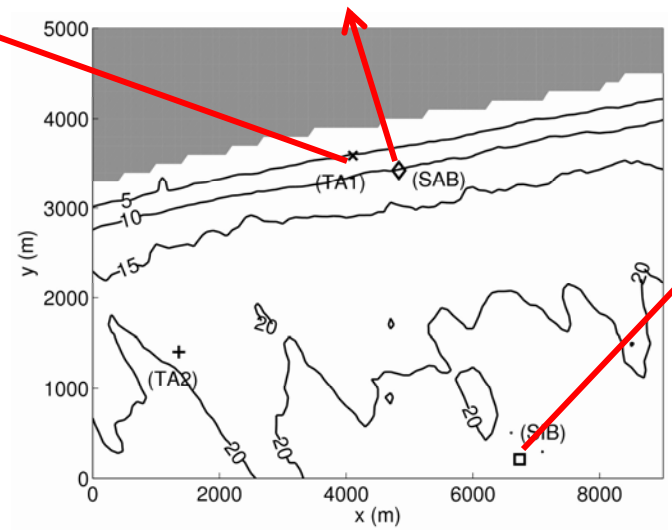


Shown as a percentage of the actual wave direction for all six cases

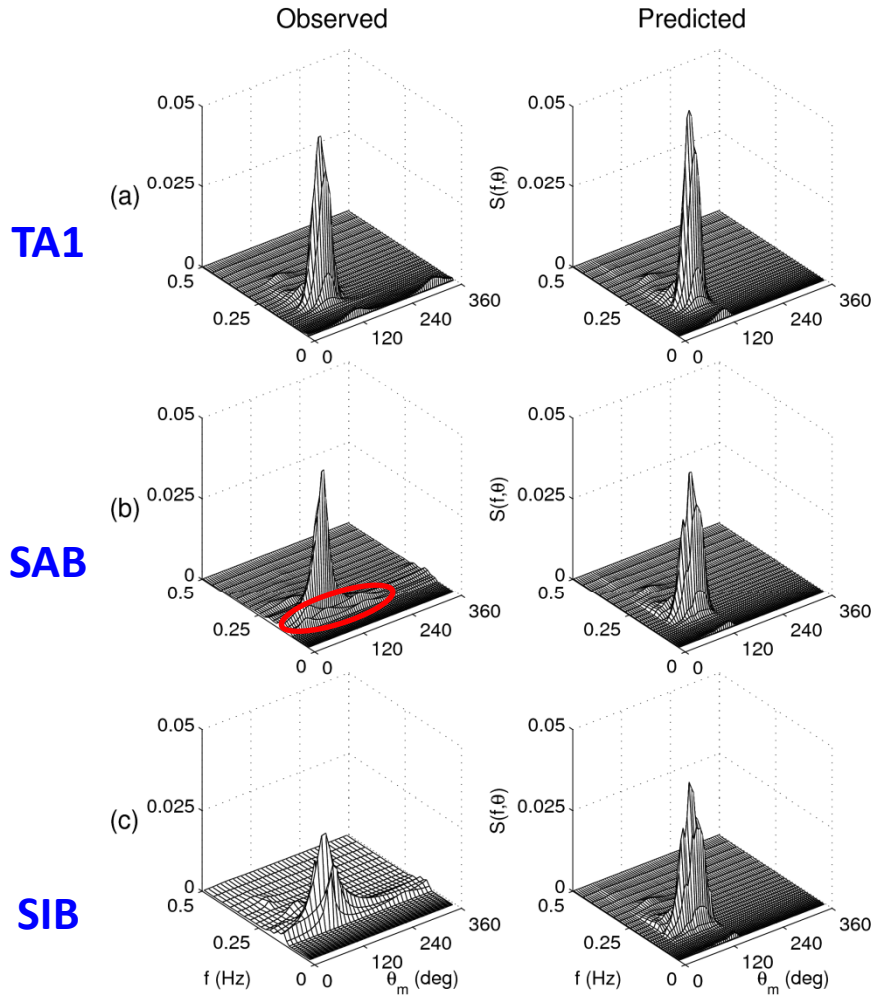
Comparison of assimilated solution versus observation



TA1 **SAB** **SIB**

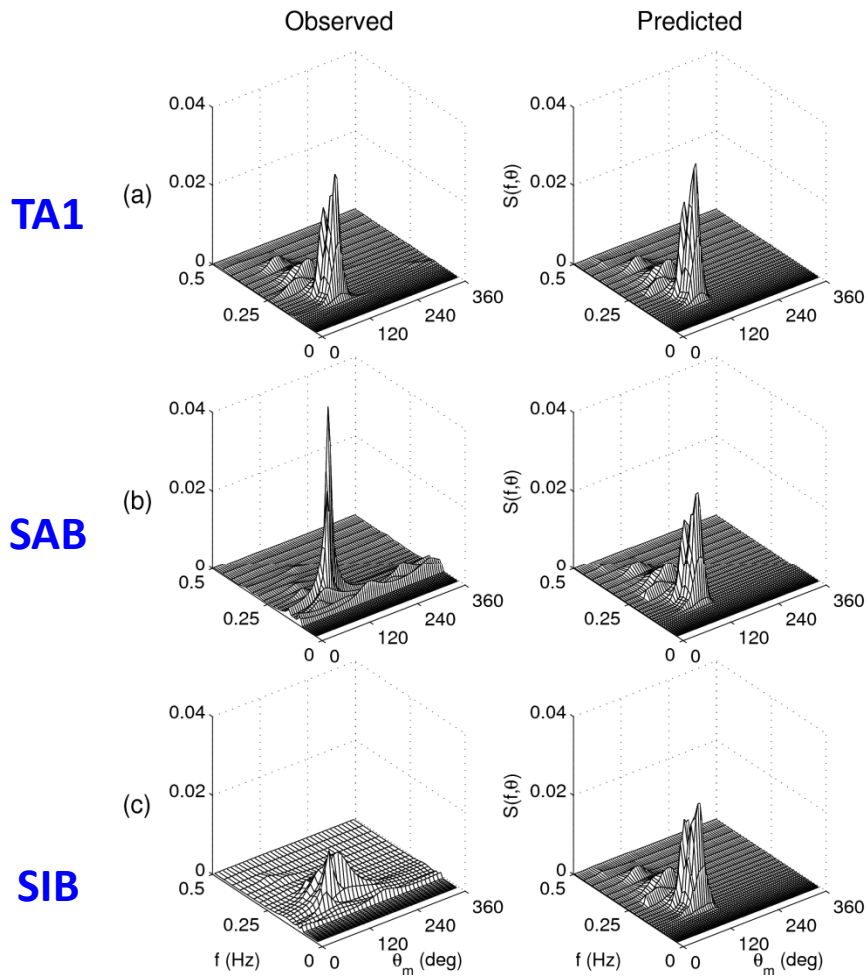


Observed versus predicted wave spectral energy density

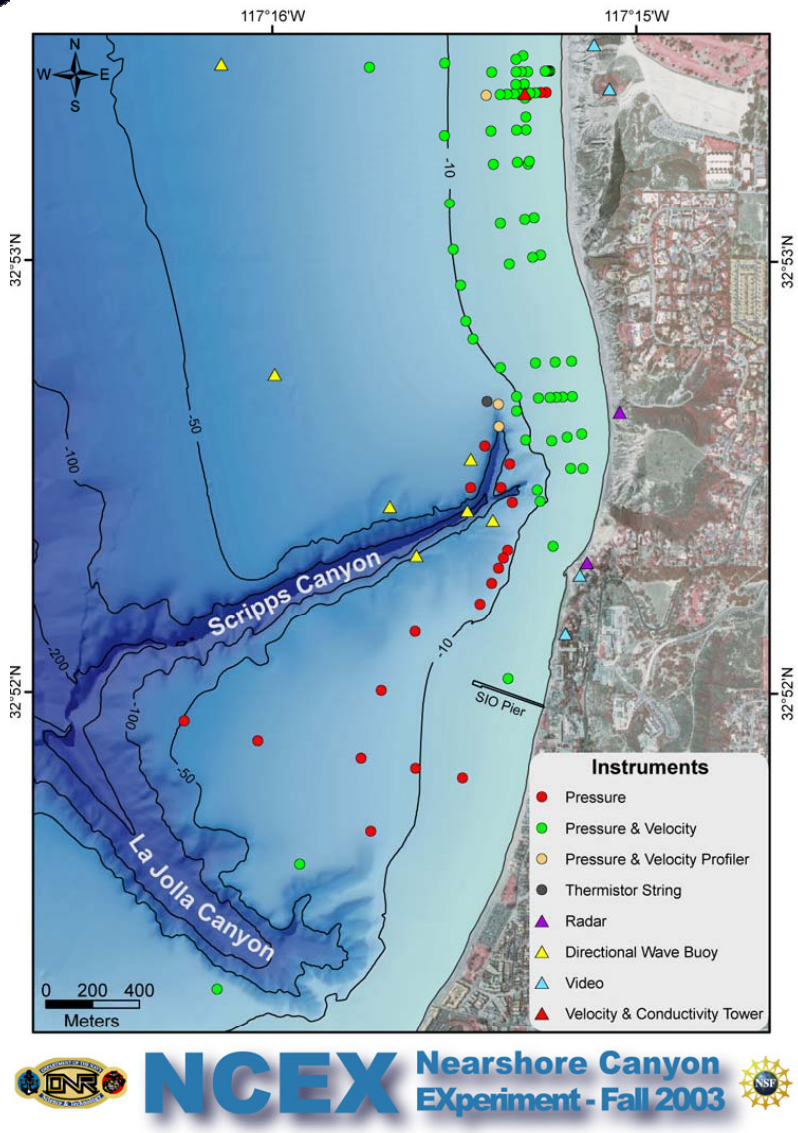


28 JAN 2009 3PM

Observed versus predicted wave spectral energy density



28JAN 2009 12PM



Nearshore Canyon EXperiment

2003 September-December

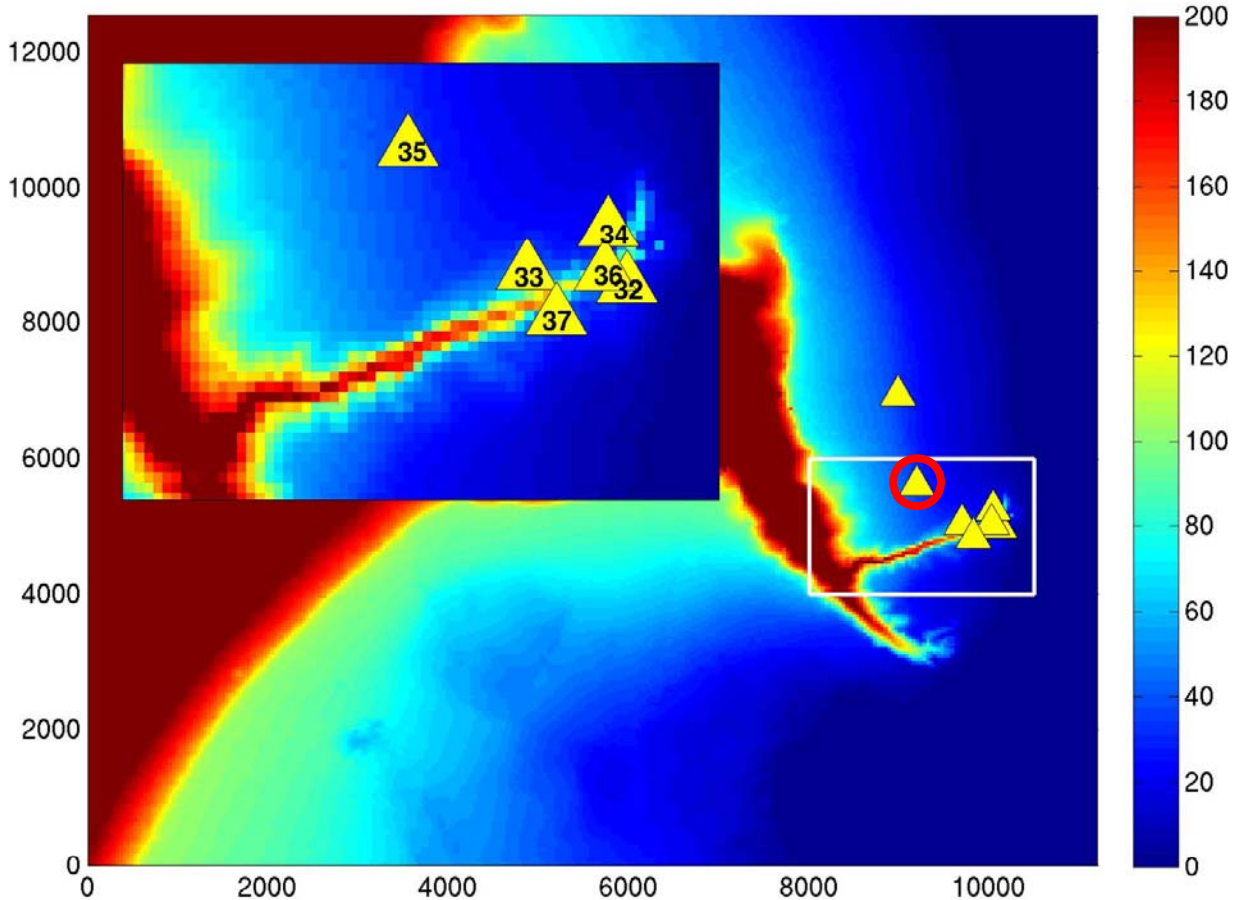
Focus: "…how abrupt coastal bathymetry affects

- Wave propagation
- Circulation
- Swash
- Morphological evolution"

http://science.whoi.edu/users/elgar/NCEX/ncex.html

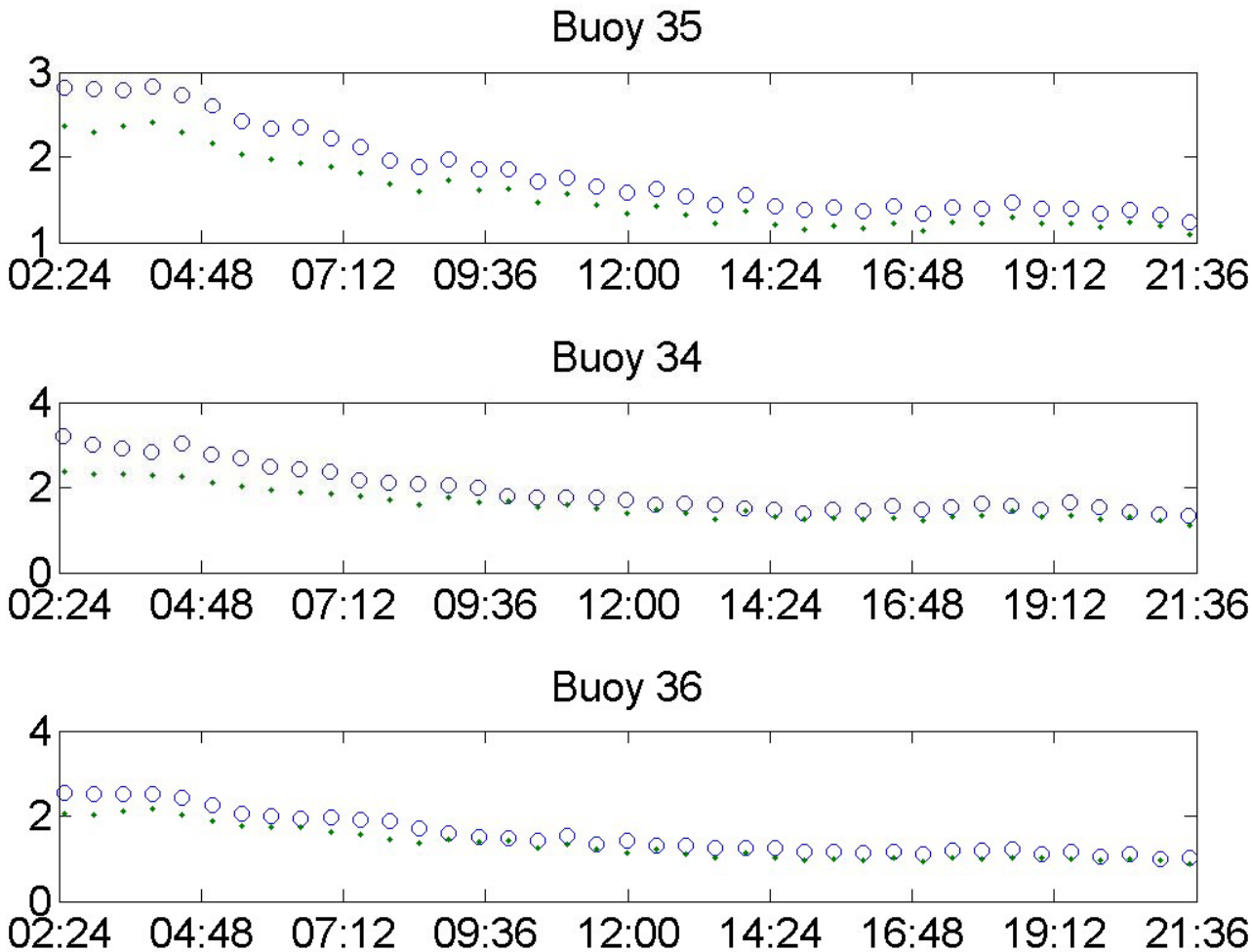
"Wave and current data were collected by T.H.C. Herbers (Naval Postgraduate School), W.C. O'Reilly (Scripps Institution of Oceanography), and S. J. Lentz (Woods Hole OCeanographic Institution) with funding from the National Science Foundation (Physical Oceanography) and the Office of Naval Research (Coastal Geosciences)."

Model Details: NCEX



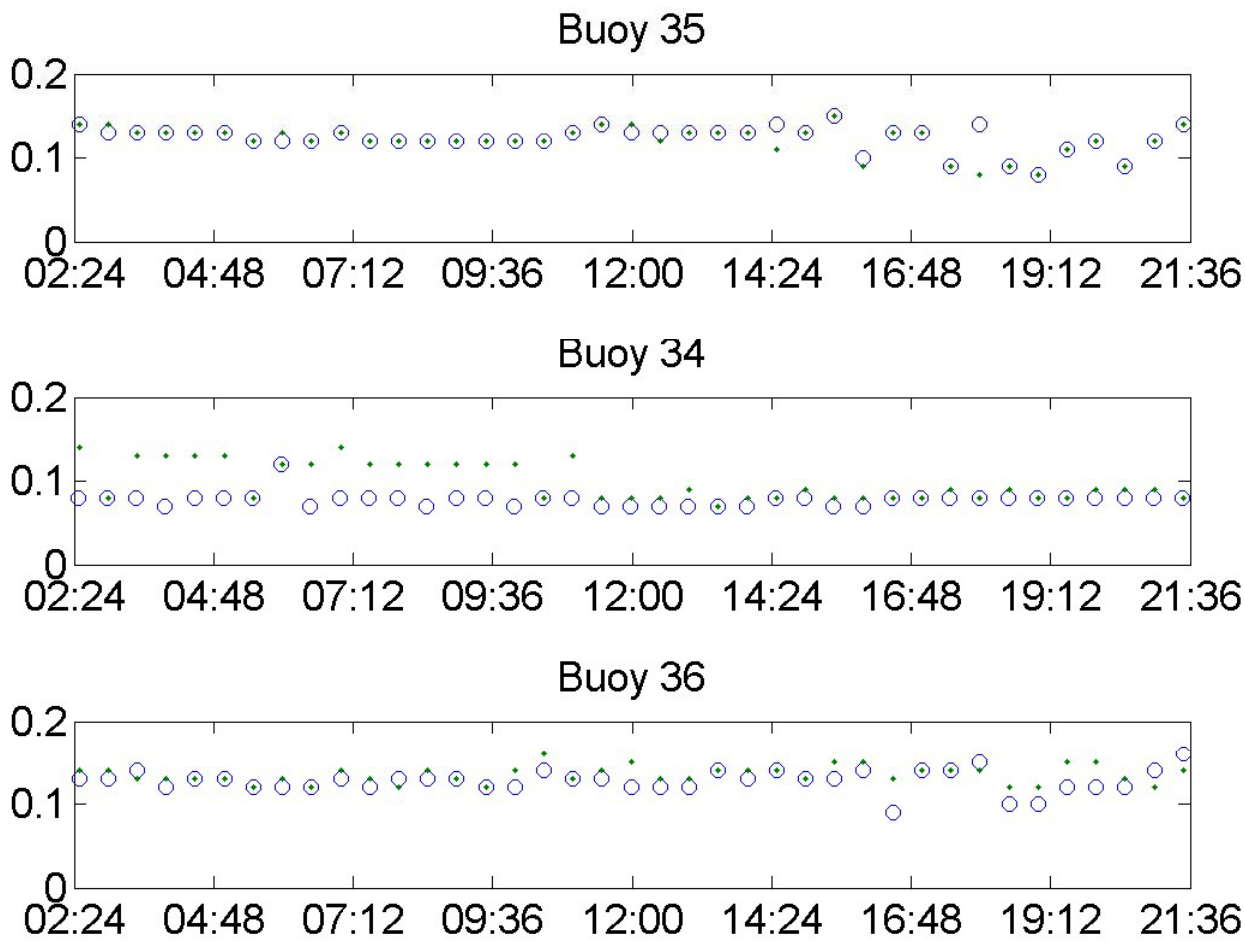
- Local coordinates in meters
- 288 grid points in x-direction; 39 meter resolution
- 271 grid points in y-direction; 46 meter resolution
- Use station 35 for assimilation

Comparison of H_{sig} (m)



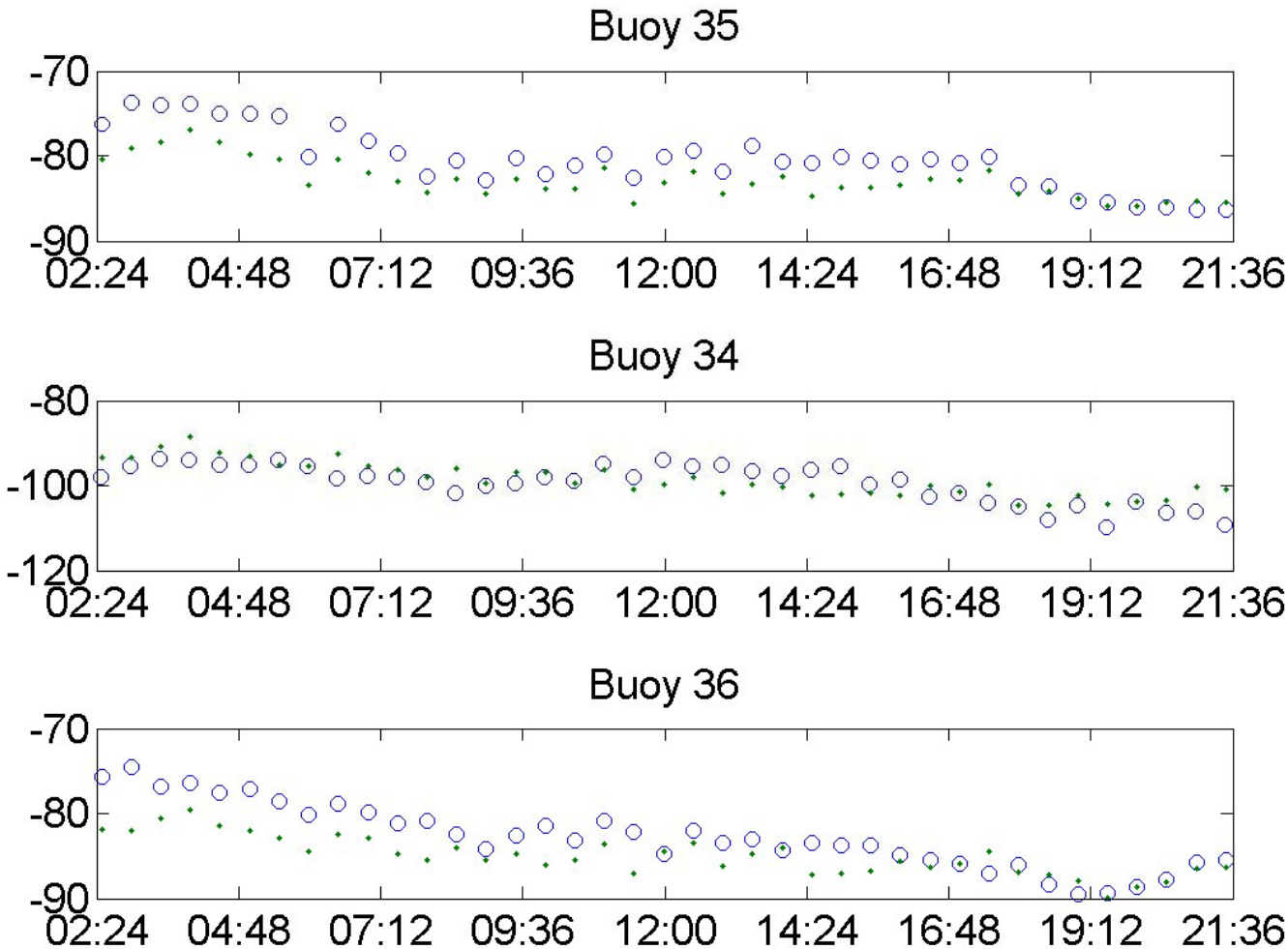
Blue circles are data, green dots are model results

Comparison of f_p (Hz)



Comparison of θ_m (deg)

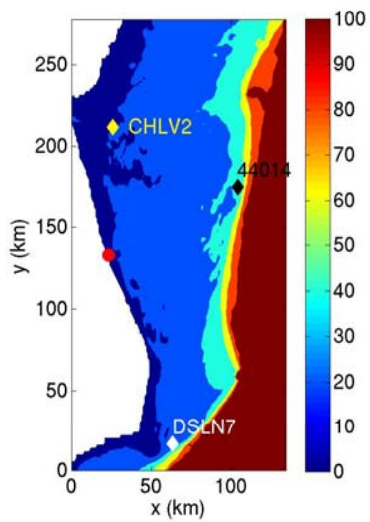
North is 0°, West is -90°



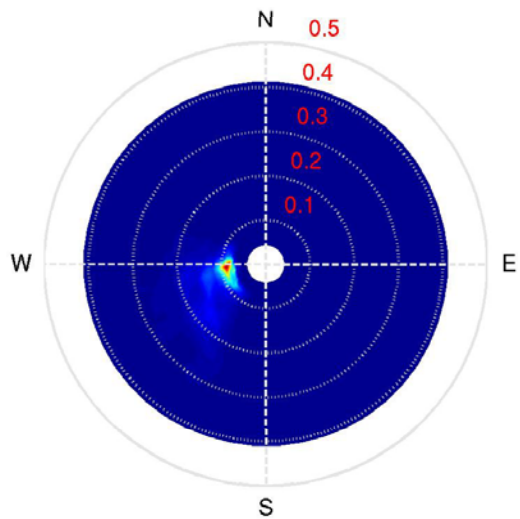
Duck, NC Studies

Assimilation minimizes errors in the offshore boundary condition and bottom friction

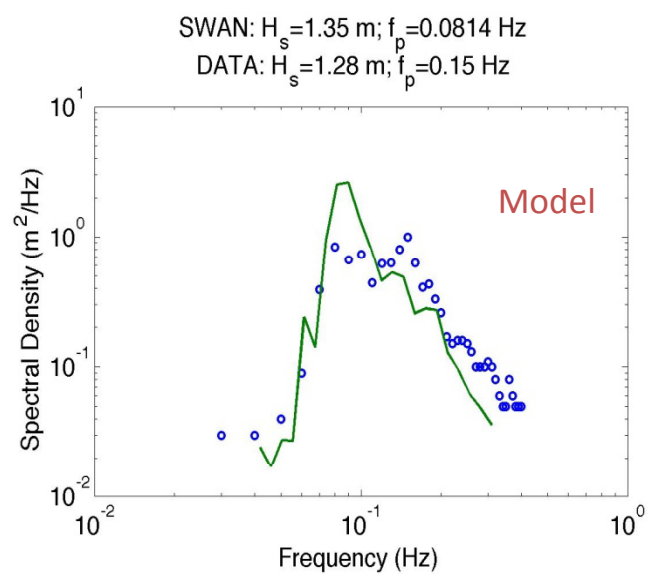
Water depth (m)



S(f,θ) (input)



Spectra at CHLV2

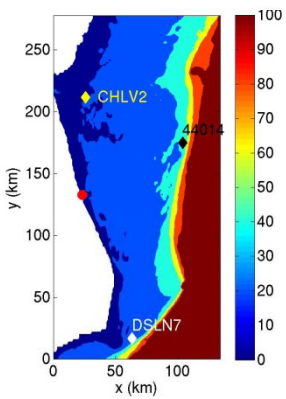


- Assimilation data location: 8m array
- Energy concentrated around one dominant frequency
- Wave mean direction near-normal to offshore boundary (westward propagation)

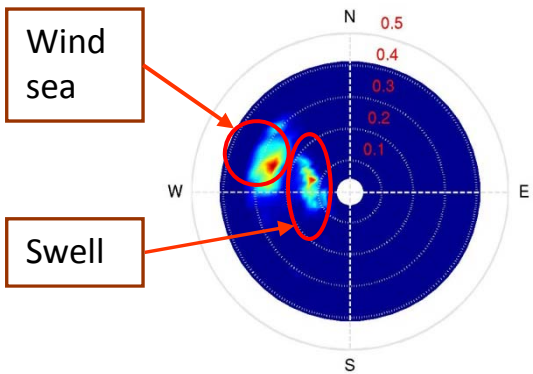
Limitations of Walker's Model

- Bi-modal distribution – wind sea & swell

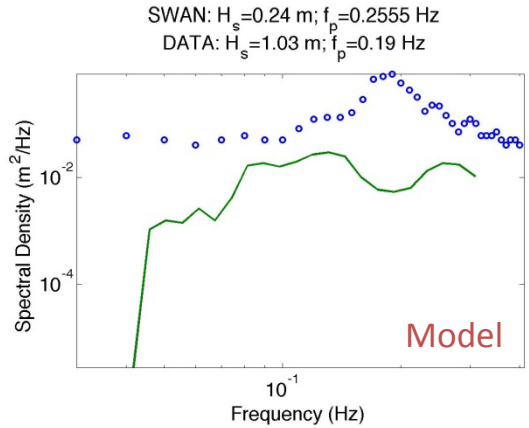
Water depth (m)



Input Spectra

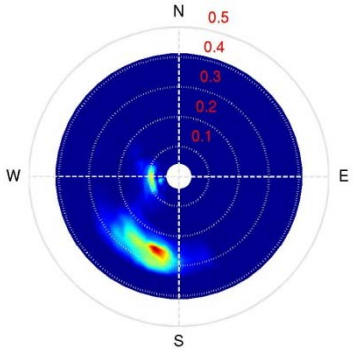


Spectra at DSLN7

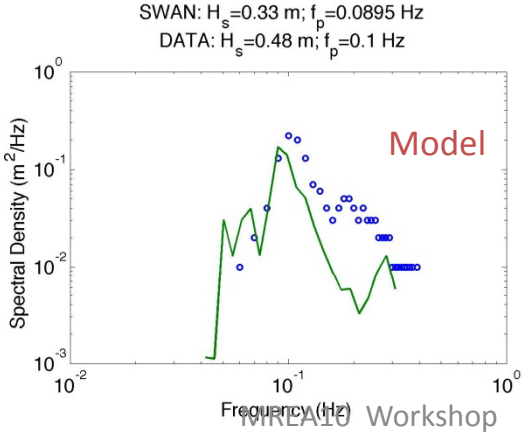


- Wave mean direction at large angle to offshore boundary

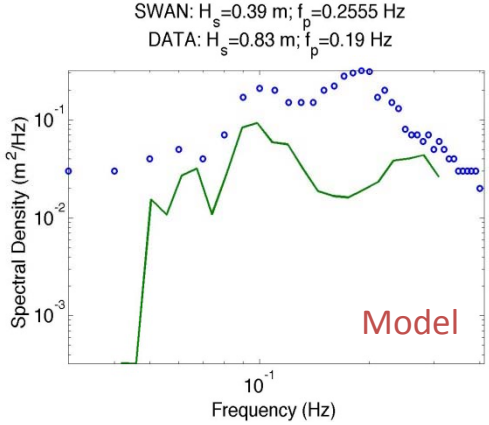
Input Spectra



Spectra at CHLV2



Spectra at DSLN7



Summary & Conclusions

- Wave data assimilation system developed by Walker (2006) has been shown to correct boundary conditions to improve wave energy prediction within computational domain.
- Assumption that propagation dominates and source terms are neglected in the adjoint model.
- Results demonstrate that assimilation system can reproduce most wave energy in the domain even when winds and associated nonlinearities are significant.
- Accuracy of assimilation system is reduced when there is relatively little energy at the assimilation location.
- The transfer of energy between frequencies as the waves propagate is not captured by the model; consequently the spectral shapes from the model are broader than the measurements.

Future Plans

- Development of new wave adjoint (FY10-12):
  - Include lateral boundaries (offshore boundary conditions not limited to offshore uniform)
  - Include all source/sink terms
  - Develop cost function to be minimized
- *Investigate* assimilation of currents into nearshore modeling systems
  - Investigate use of OPENDA and NCODA
  - Planned trials with SHARC Wave Glider in 2011
    - Possible use of ADCP currents for hydrodynamic nowcast

In Memoriam
Y Larry Hsu (April 6,1947 – July 30, 2010)

Larry was recognized as a leading researcher in the field of surf forecasting. Implemented several physics-based improvements to the Navy Standard Surf Model (NSSM), including roller option, which he transitioned to the Naval Oceanographic Office. Larry performed numerous validation studies with Delft3D. Validation studies showed that Delft3D was robust and accurate in predicting nearshore wave height and flows in a depth-averaged environment. He also worked with industry in the development of an Iridium-based directional wave buoy with an Acoustic Doppler Current Profiler.

Acknowledgments

- Thanks to Dr. David Walker (SRI International) for sharing his SWAN model data assimilation code and working with NRL in testing and evaluating using directional wave spectra.

Questions ?