Inversion for water column and sediment sound speed profiles in range-dependent environments with application to data from the SW06 Experiment

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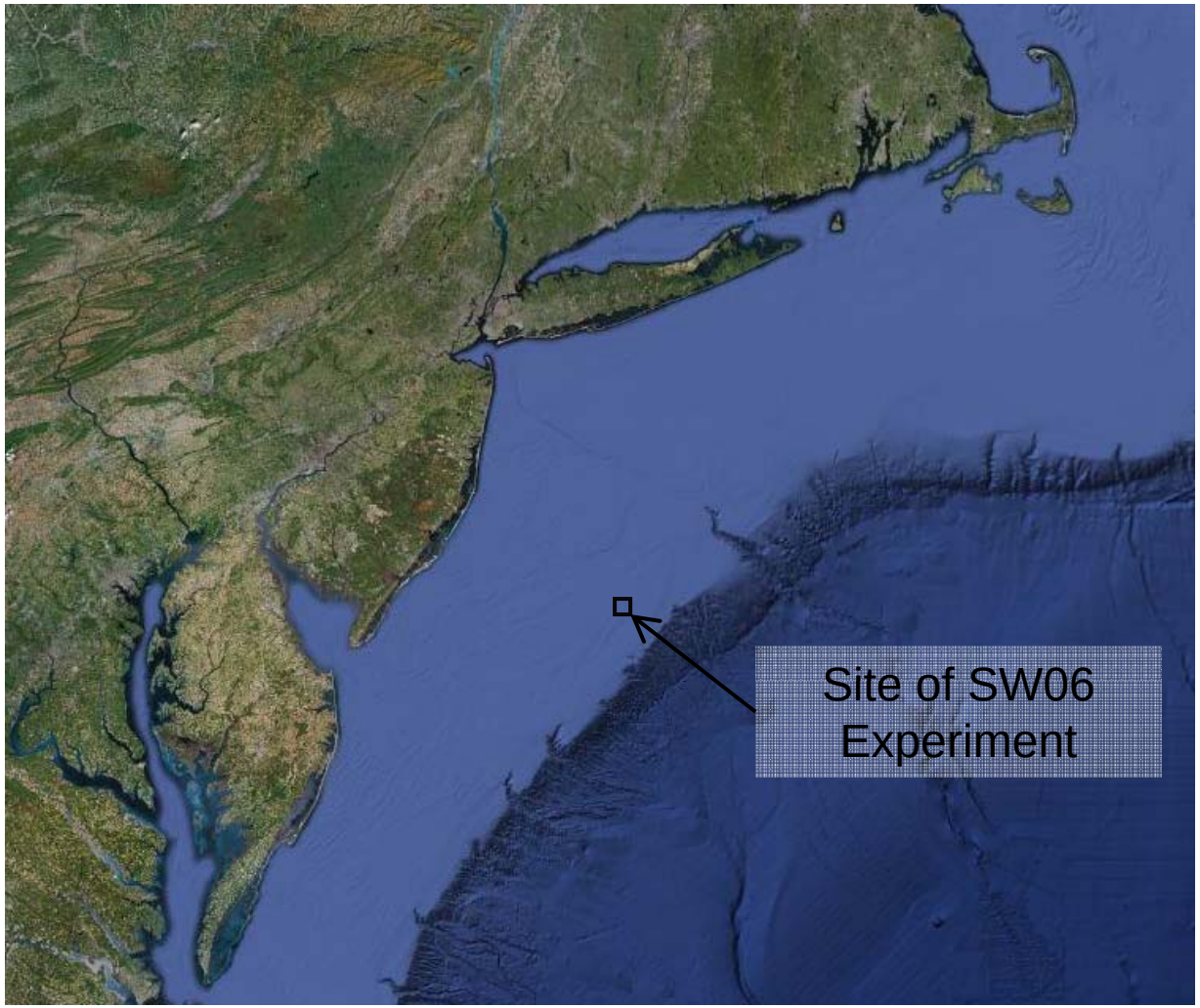
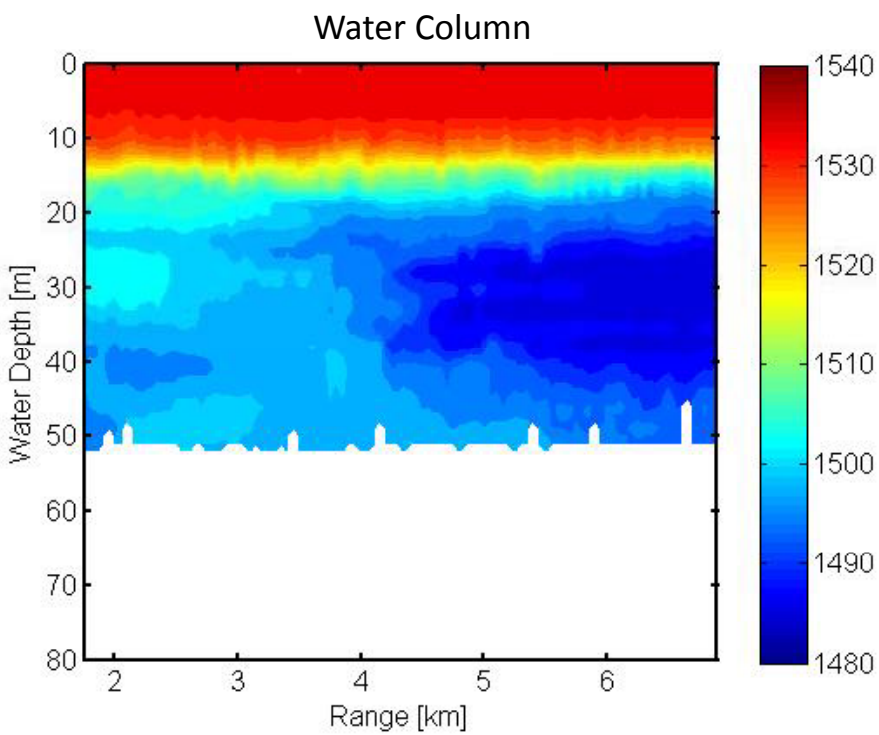
Shallow Water 2006 Experiment

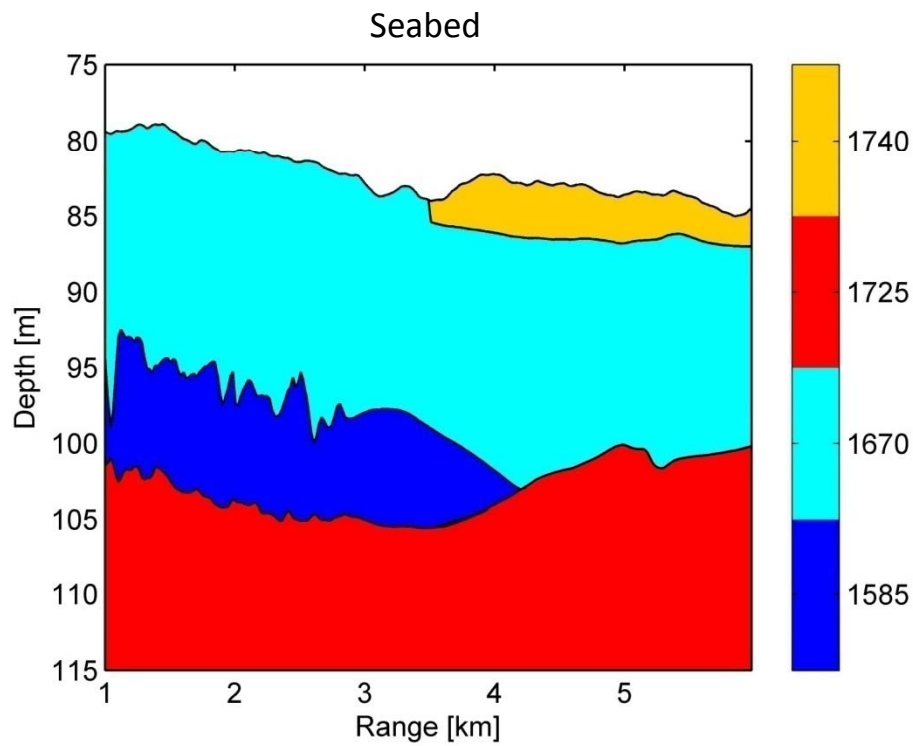


Objectives

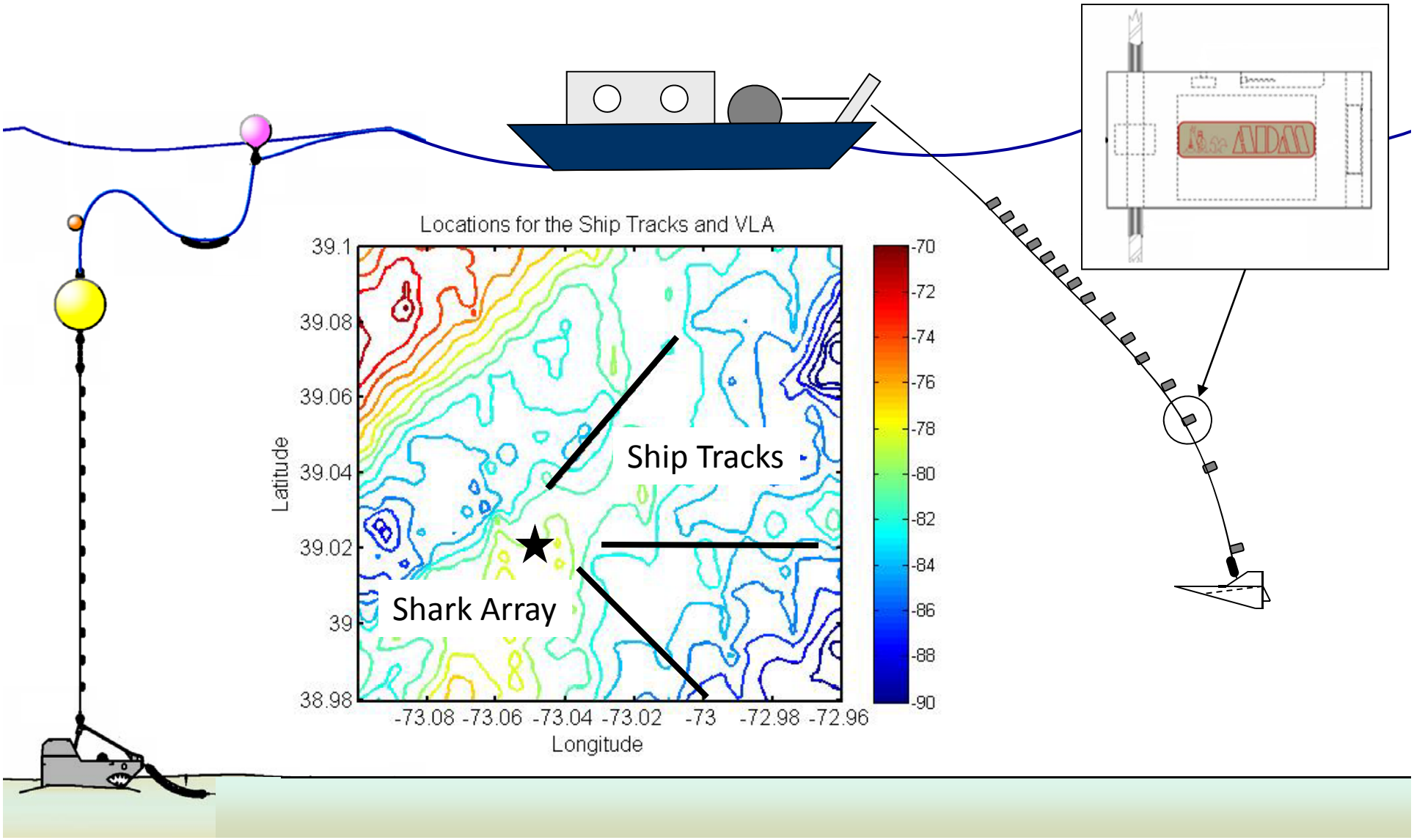
This work is concerned with developing inverse methods to estimate sound speed profiles of the water column and seabed. These methods are designed to

- Manage complications caused by the high spatial and temporal variability of the shallow-water environment
- Address the task of decoupling of the seabed and water column inverse problems





SW06 Experiment



Wave Number Estimation

The Hankel Transform Pair using the far field approximation

$$p(r;z,z_0)=\frac{e^{-i\pi/4}}{\sqrt{2\pi r}}\int_{-\infty}^{\infty} g(k_r;z,z_0)\sqrt{k_r}\,e^{ik_r r}\,dk_r$$

$$g(k_r;z,z_0)=\frac{e^{i\pi/4}}{\sqrt{2\pi k_r}}\int_{-\infty}^{\infty} p(r;z,z_0)\sqrt{r}\,e^{-ik_r r}\,dr$$

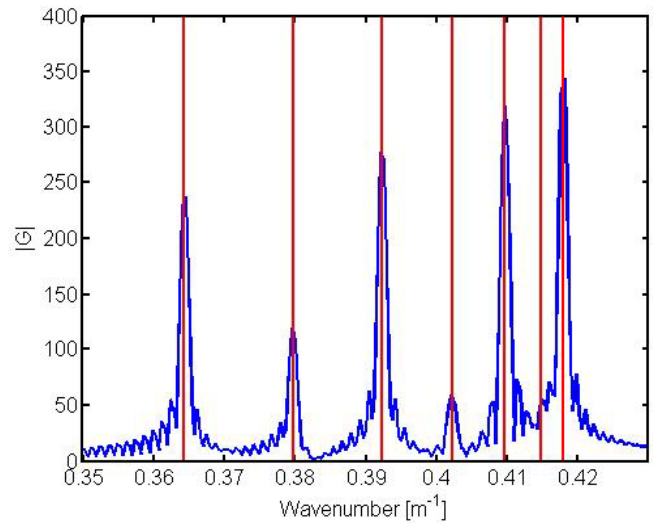
The Short-Time Fourier Transform (STFT)

$$g(k_r;\hat{r},z,z_0)=\frac{e^{i\pi/4}}{\sqrt{2\pi k_r}}\int_{-\infty}^{\infty} w_L(r;\hat{r})\,p(r;z,z_0)\sqrt{r}\,e^{-ik_r r}\,dr$$

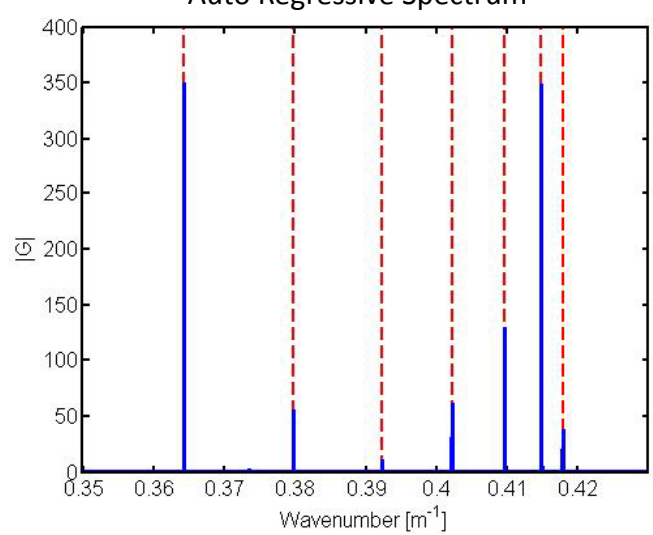
Auto Regression (AR)

$$x_n=\sum_{k=1}^{p} a_k x_{n-k} \longrightarrow P_{AR}=\frac{\sigma^2 T}{\left|1+\sum_{k=1}^{p} a_k e^{-i2\pi fkT}\right|^2}$$

Hankel Transform Estimate



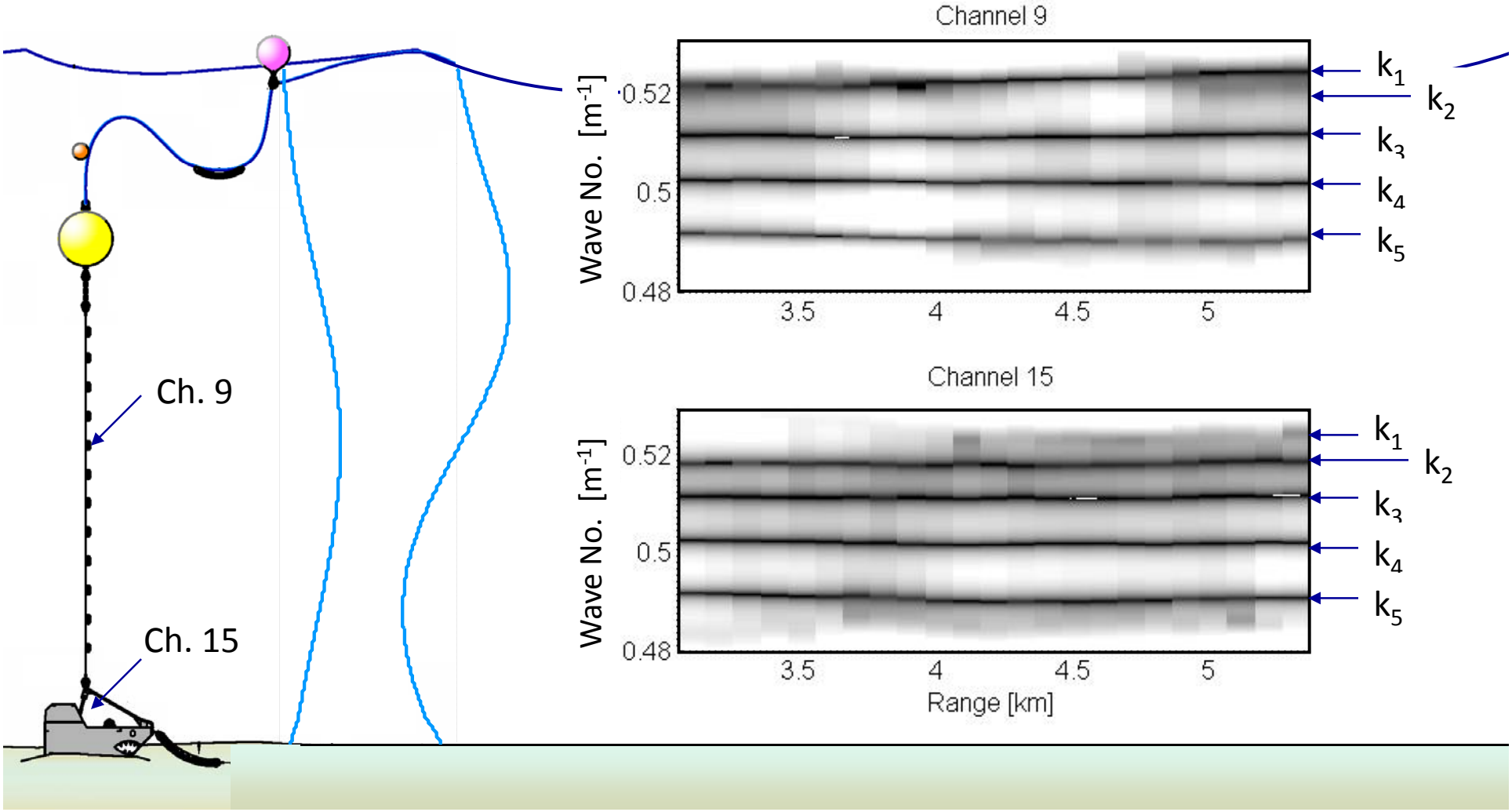
Auto Regressive Spectrum



Estimated Wave Numbers

Along Shelf Track

125 Hz Data: AR estimator using 1600 meter sliding window with 95% overlap



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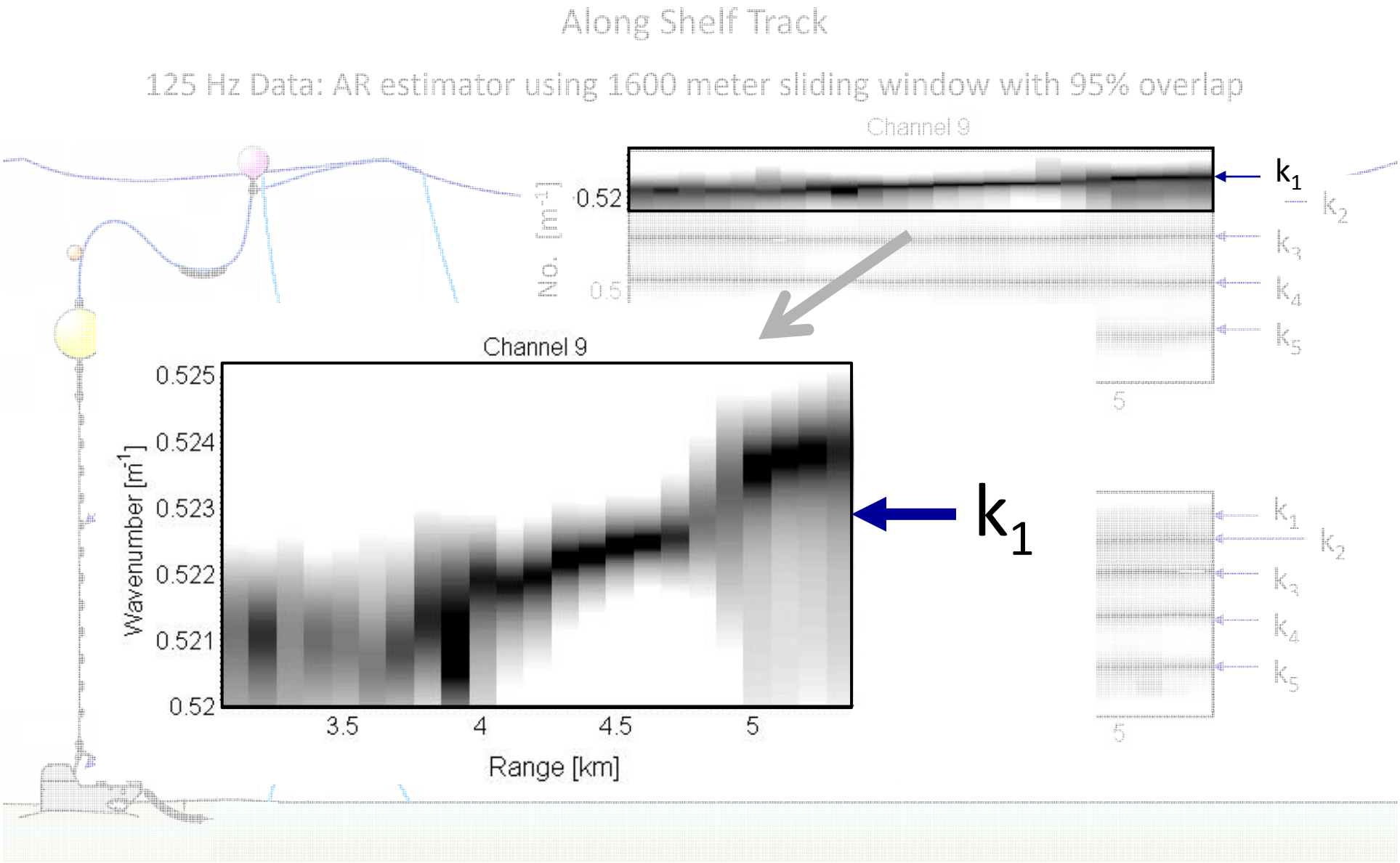


Perturbative Inversion

A relation between a perturbation to sound speed and a perturbation to horizontal wave numbers is formulated from the depth separated normal mode equation:

$$\Delta k_n = \frac{1}{k_n} \int_0^\infty \rho^{-1}(z) Z_n^2(z) k^2(z) \frac{\Delta c(z)}{c_0(z)} dz$$

This equation can be written in the form of a Fredholm integral of the first kind:

$$d_i = \int_0^\infty m(z) G_i(z) dz \qquad i = 1, \ldots, N$$

Which can be written in matrix form as:

$$\mathbf{d} = \mathbf{G}\mathbf{m}$$

$\mathbf{d}$ is a vector representing the data

$\mathbf{G}$ is a matrix representing the forward model (kernel)

$\mathbf{m}$ is a vector representing the model parameter

Approximate Equality Constraints

used to constrain the water column inverse problem

Find a solution that satisfies the data: $\mathbf{Gm} = \mathbf{d}$

Relative Equality Constraint: $\mathbf{Lm} = \mathbf{0}$

Absolute Equality Constraint: $\mathbf{Am} = \boldsymbol{\alpha}$

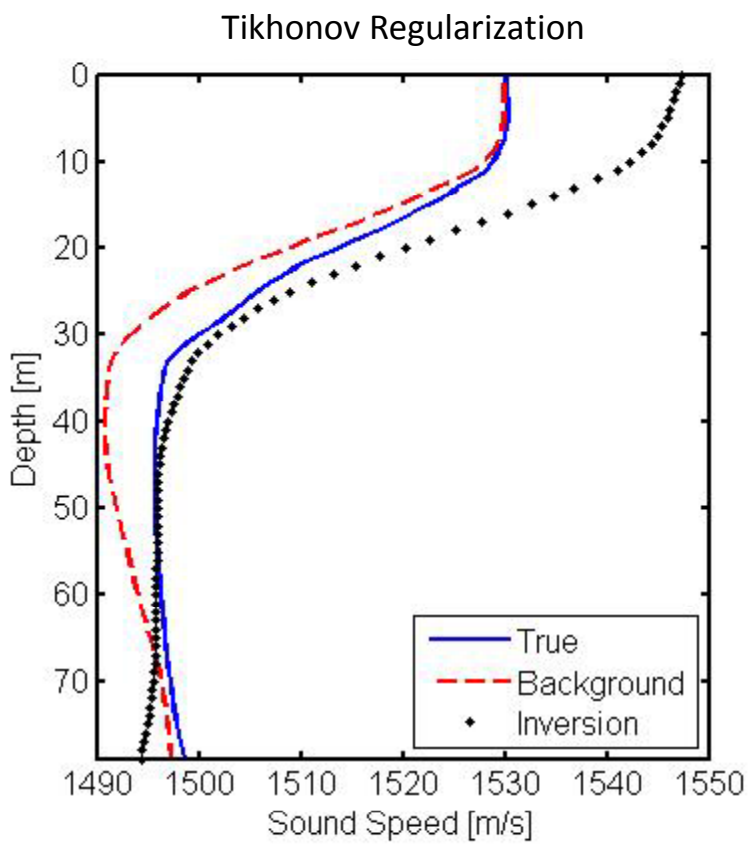
The *relative equality constraint* is the well known smoothness constraint of Tikhonov regularization.

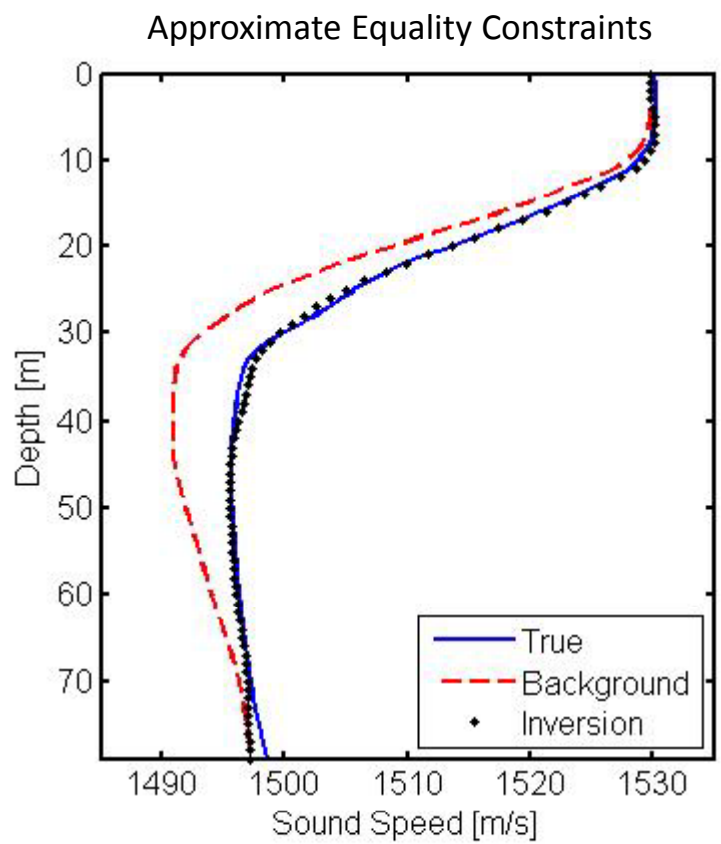
The *absolute equality constraint* is chosen to restrict perturbations from the background profile near the sea surface and seafloor.

The matrix $\mathbf{A}$ specifies where to apply the absolute equality constraint.
The vector $\boldsymbol{\alpha}$ specifies the value the solution should take at these points.

The solution is given by: $\hat{\mathbf{m}} = (\mathbf{G}^T\mathbf{G} + \lambda_1^2\mathbf{L}^T\mathbf{L} + \lambda_2^2\mathbf{A}^T\mathbf{A})^{-1}\mathbf{G}^T\mathbf{d}$

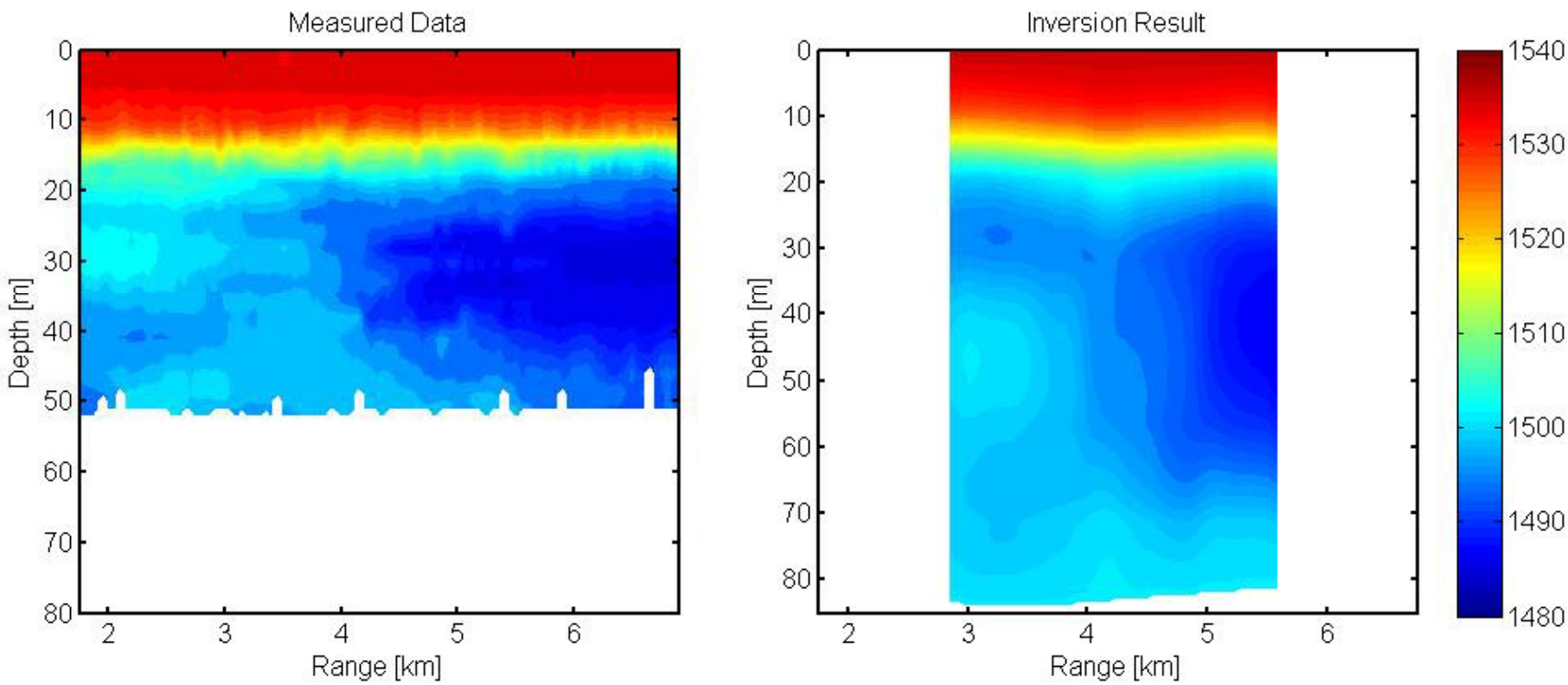
Comparison: Tikhonov Regularization and Approximate Equality Constraints





Application to SW06

Inversion algorithm applied to run 8 from the SW06 experiment.
Results obtained using low order wave number data from the 125 and 175 Hz signals.



Ballard and Becker, "Inversion for water column sound-speed profiles on the New Jersey shelf using a linearized perturbative technique", *Journal of the Acoustical Society of America*, **127**(6), 2010.

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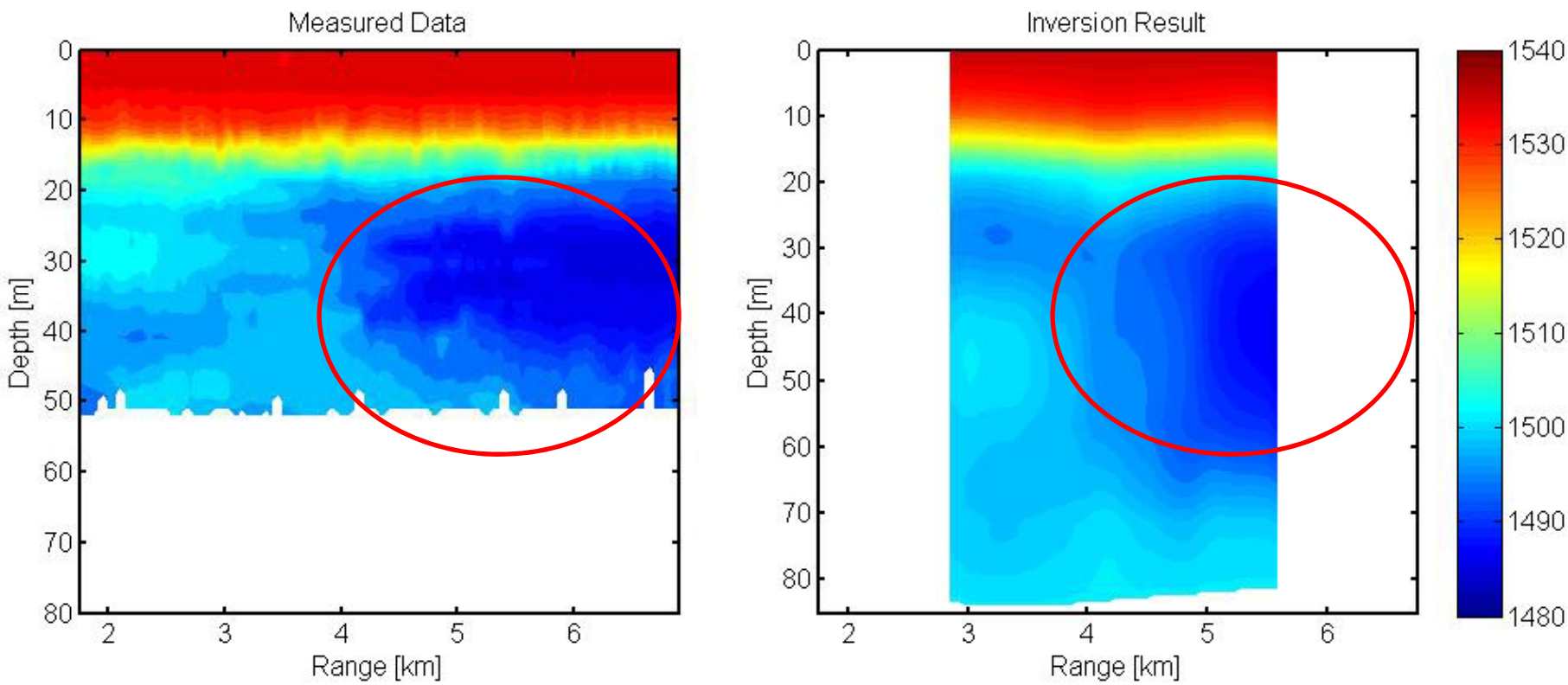

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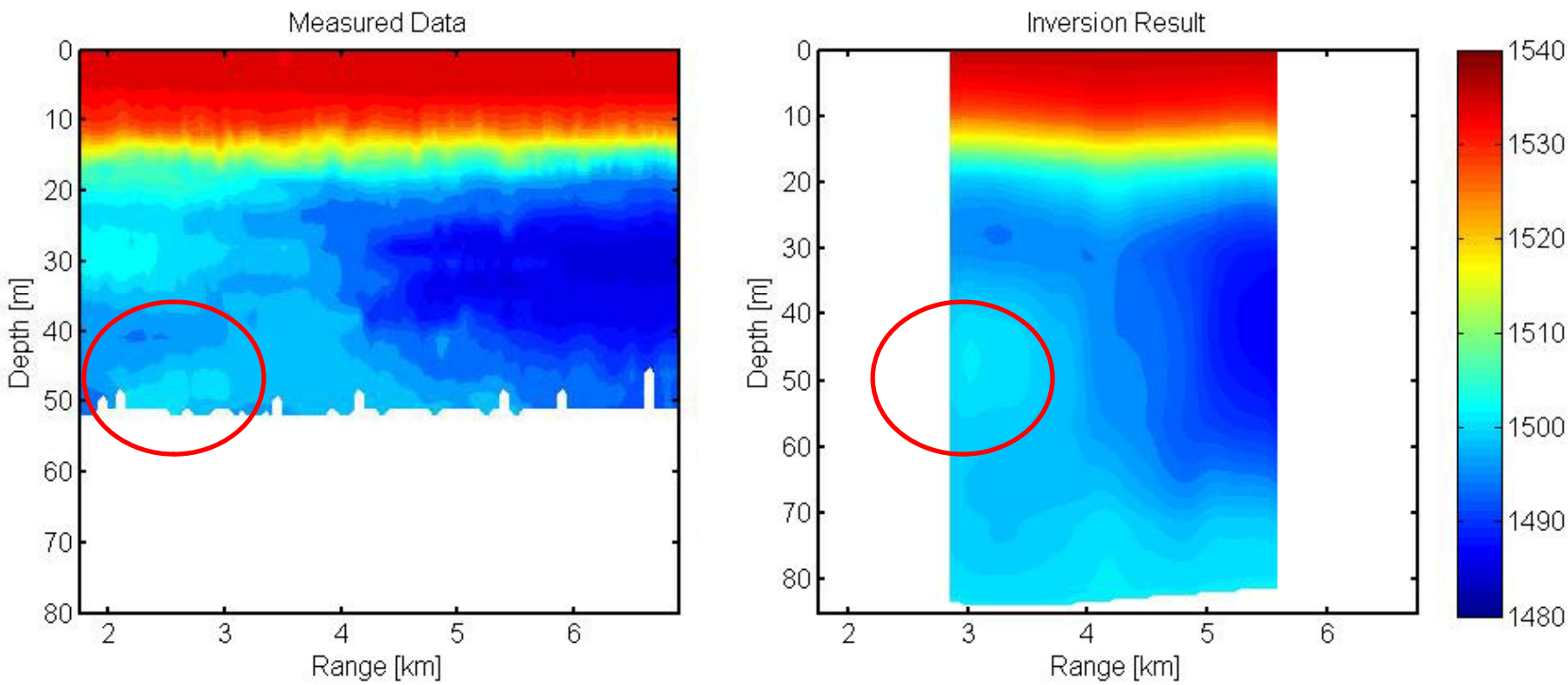

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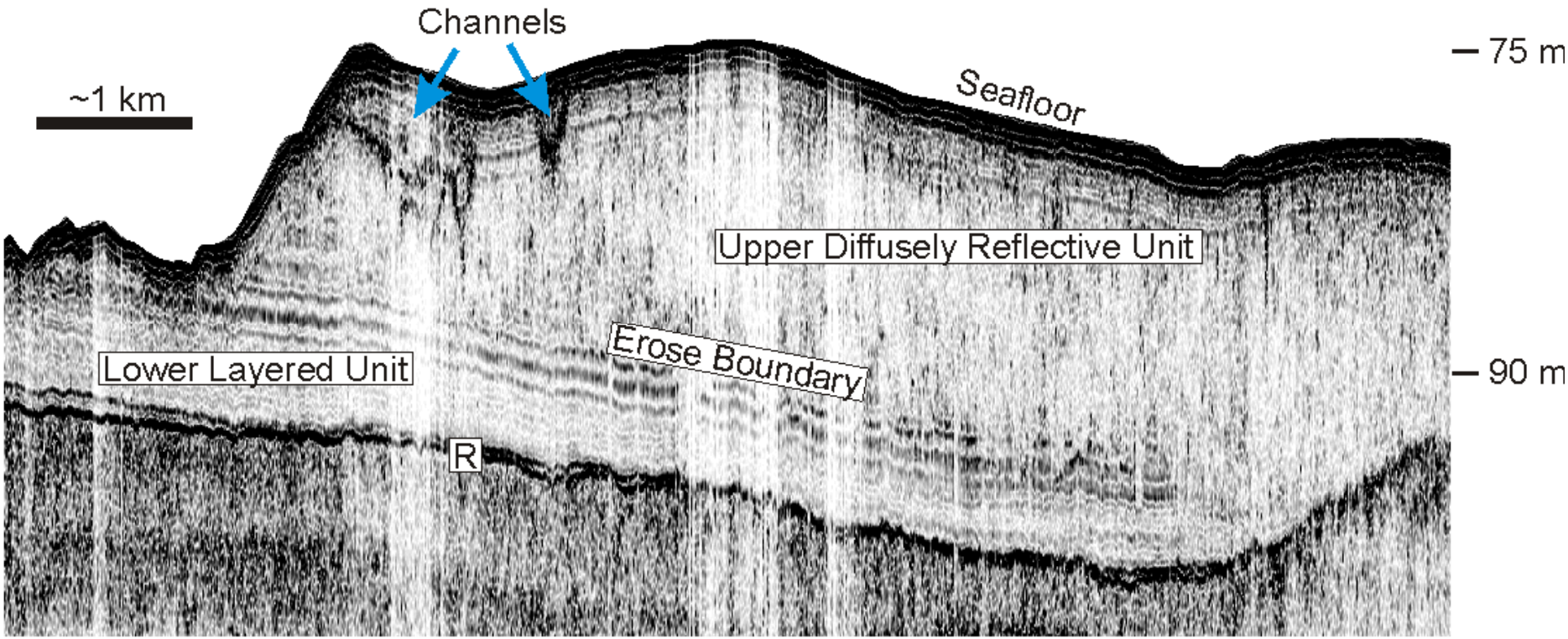
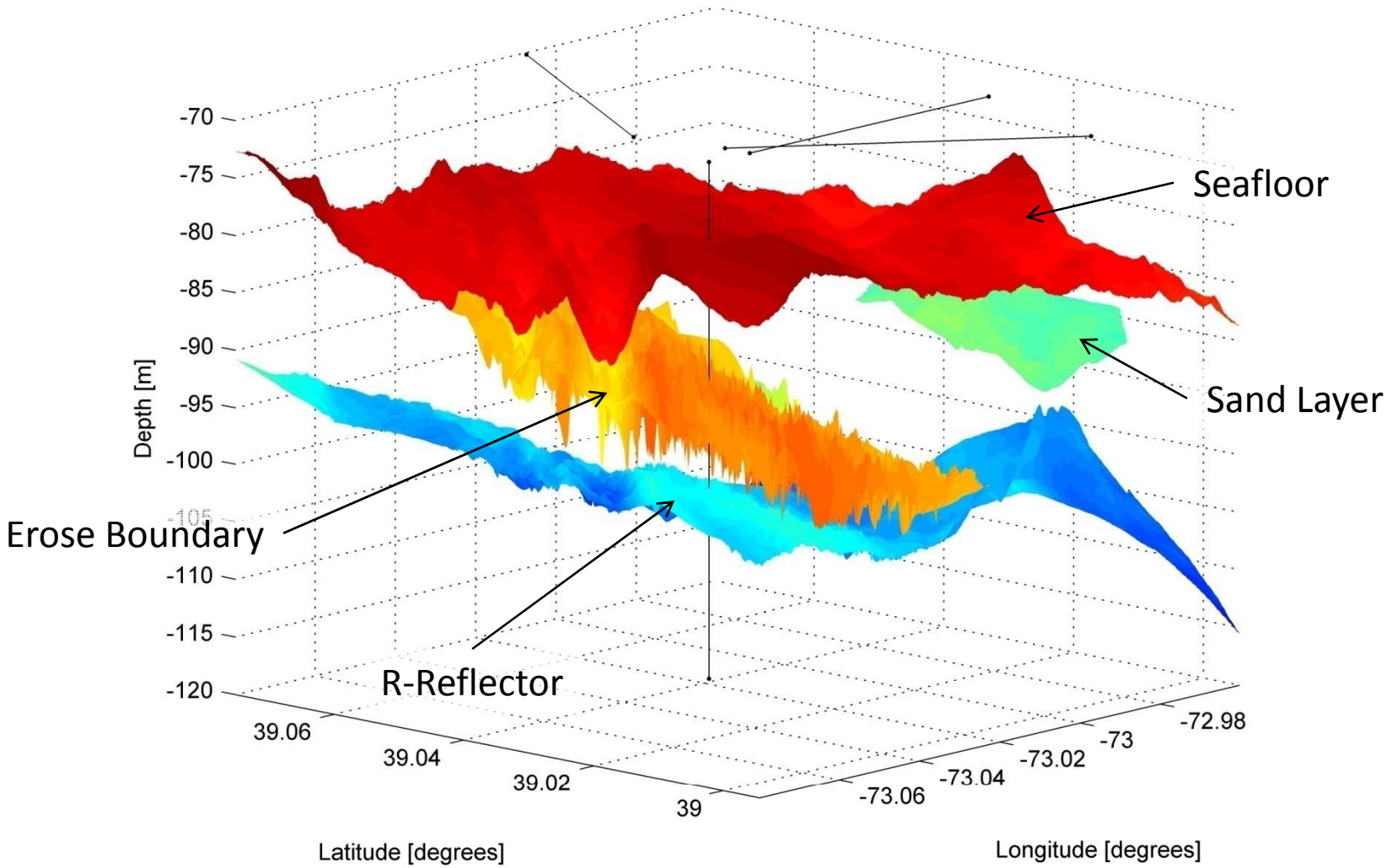
The New Jersey Shelf

chirp seismic reflection data



Chirp data provided by John Goff

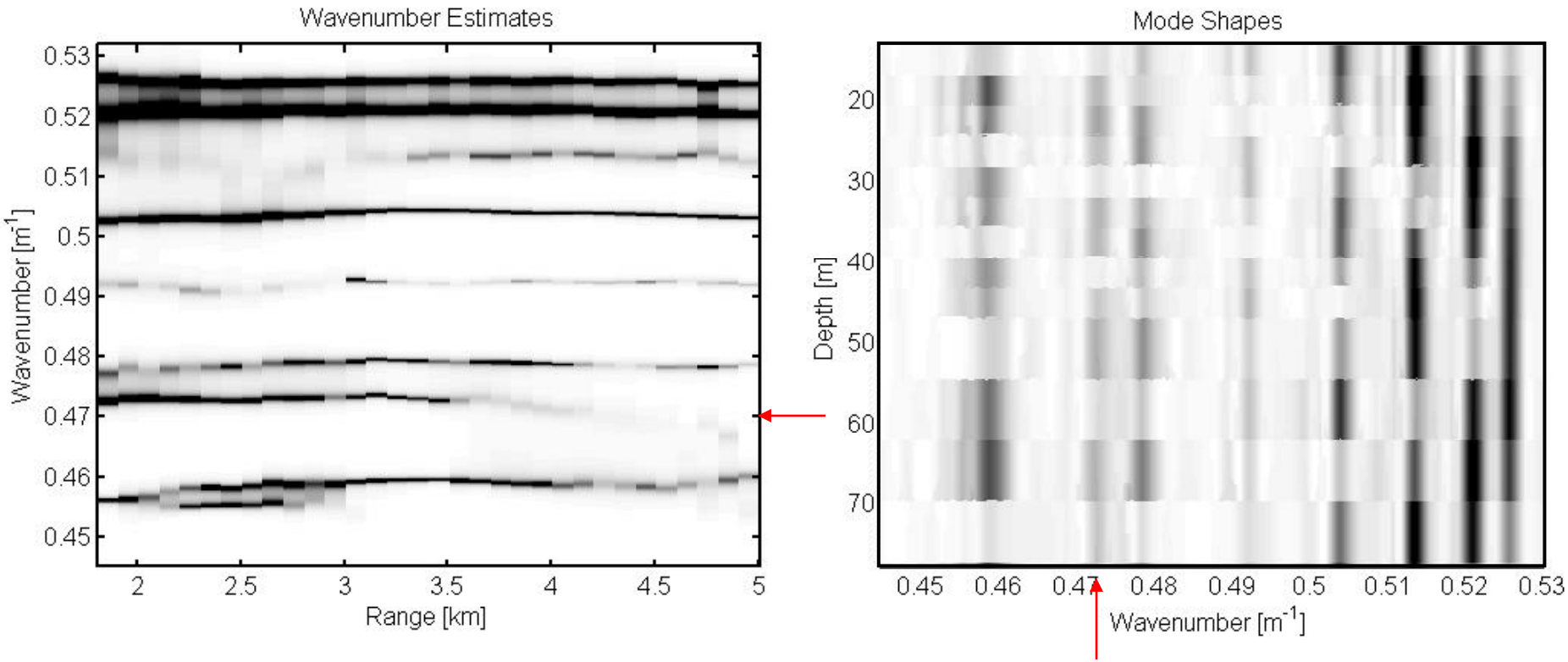
Three Dimensional Model



Wavenumber Estimates

Oblique Shelf Track

Range dependent wavenumbers estimated using auto regression with 2km aperture
Mode shapes estimated using Hankel transform on full 5km of data



Mode seven is a resonant mode which is sensitive to the low speed layer.

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Qualitative Regularization

Choose the solution that best fits some prior knowledge.

Find a solution that satisfies the data: $\mathbf{Gm} = \mathbf{d}$

and the constraint: $\mathbf{L}_q\mathbf{m} = \mathbf{0}$

where $\mathbf{L}_q$ is created by the user.

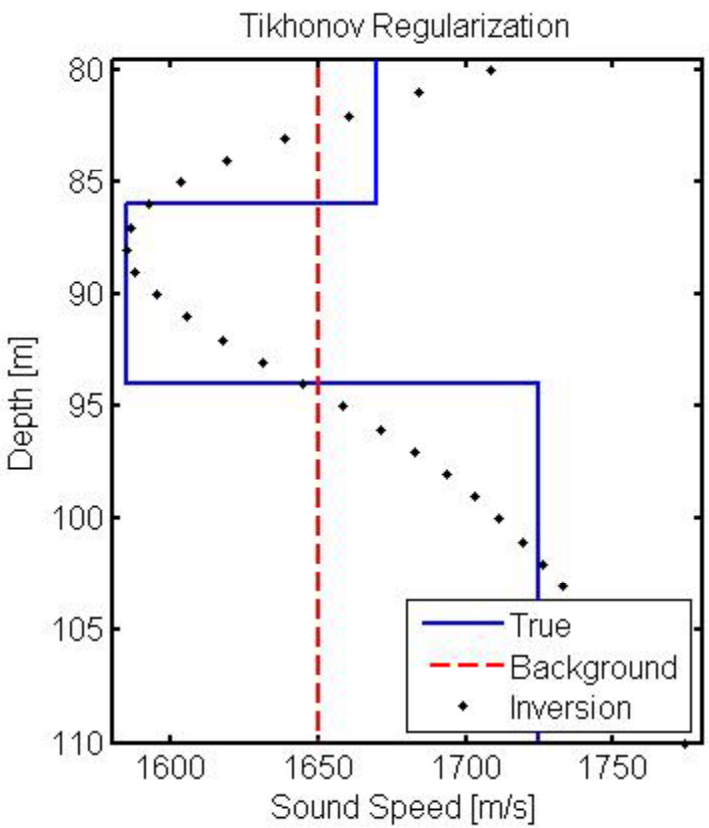
$\mathbf{L}_q$ is given by: $\mathbf{L}_q = \mathbf{L}(\mathbf{I} - \sum_{i=1}^{r} \mathbf{q}_i \mathbf{q}_i^T)$

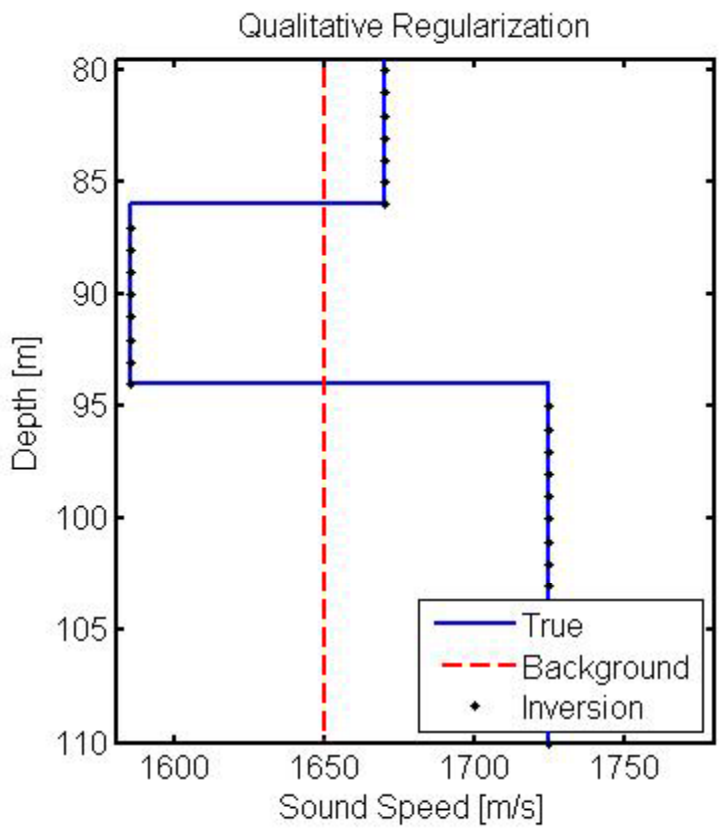
where the set $\{\mathbf{q}_i\}_{i=1}^{r}$ is on orthogonal basis for $\mathbf{Q}$.

The *qualitative regularization* solution is given by:

$$\hat{\mathbf{m}} = (\mathbf{G}^T\mathbf{G} + \lambda^2 \mathbf{L}_q^T \mathbf{L}_q)^{-1} \mathbf{G}^T \mathbf{d}$$

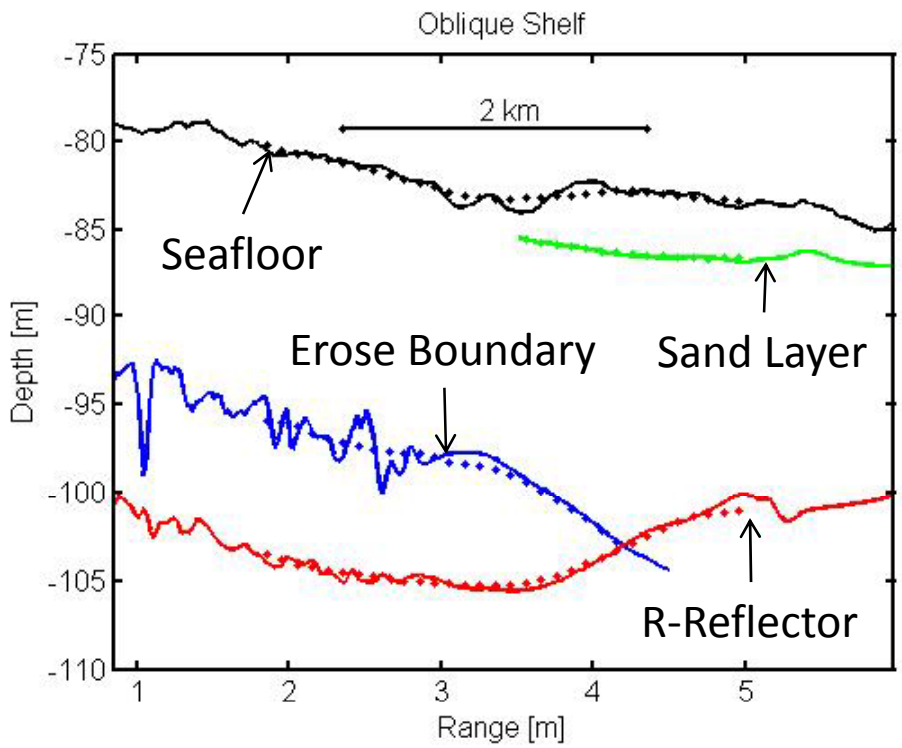
Comparison: Tikhonov and Qualitative Regularization

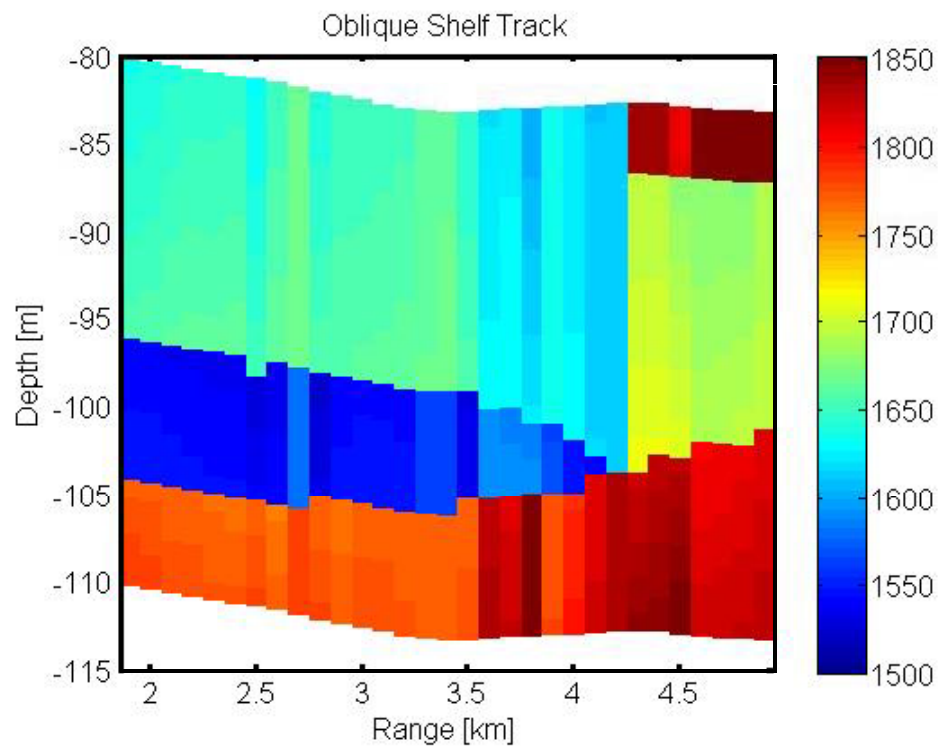




Inversion Results

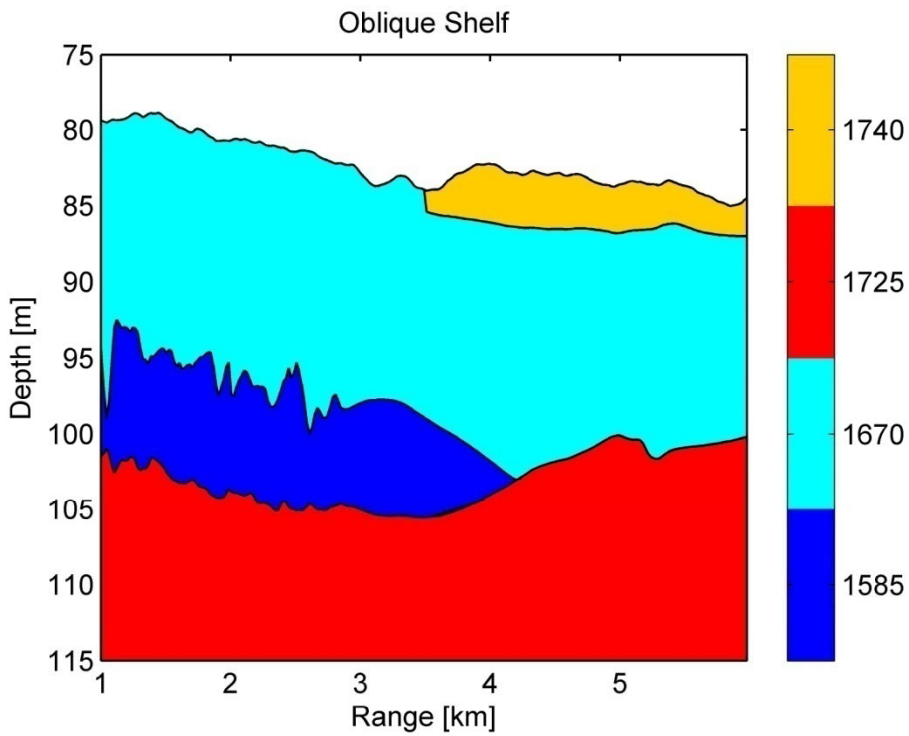
Input data to the inversion scheme: Over lapping regions are inconsistent due to inexact wave number estimates. Inaccuracies of the estimates are caused by range dependence of the environment within the estimation aperture.

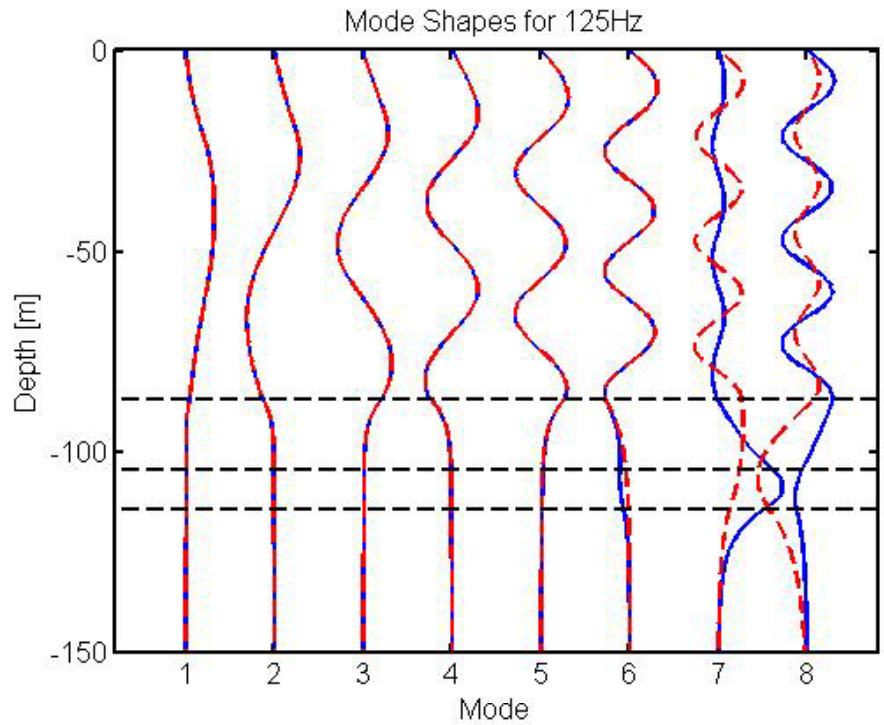




Ballard, Becker and Goff, "Geoacoustic inversion on the New Jersey shelf: three dimensional sediment model", *IEEE JOE*, **35**(1), 2010.

Inversion Result





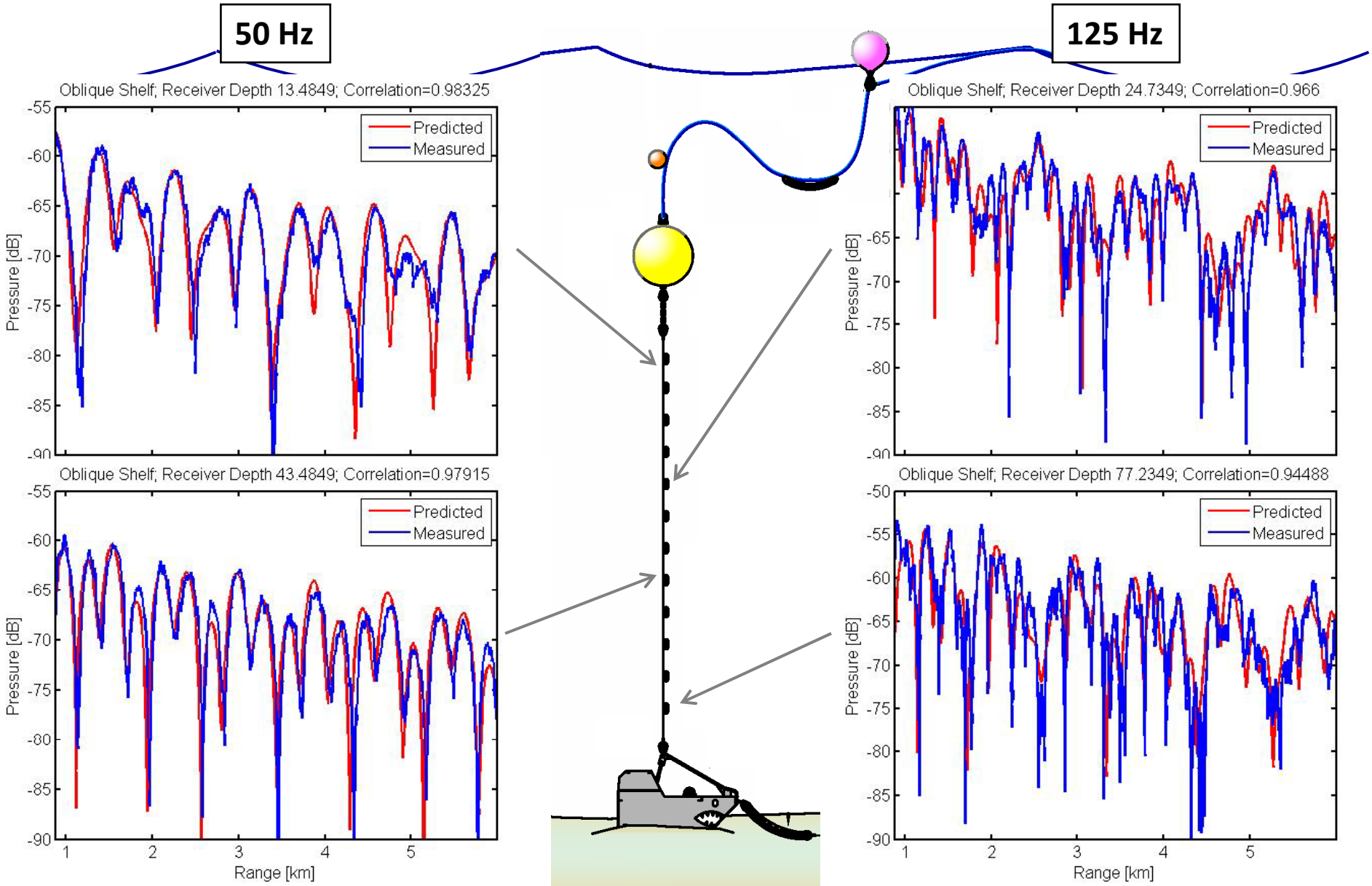
Layer	Inversion Result	Core Data
Upper Unit	1670 +/- 12 m/s	1653 +/- 34 m/s
Lower Unit	1585 +/- 19 m/s	1638 +/- 56 m/s
Below R	1725 +/- 15 m/s	1850 +/- ? m/s
Sand Ridge	1740 +/- 37 m/s	-

Low speed layer causes resonant modes

With Low Speed Layer (Ranges 1-4 km)

Without Low Speed Layer (Ranges 4-5 km)

Evaluation of Inversion Result



Summary & Conclusions

- Water Column Inversion
  - Wave number tracking of mesoscale water column variability
  - Inversion results agree well with measured data
- Seabed Inversion
  - Resonant mode sensitive to a low speed layer in the seabed
  - Inversion results predict measured acoustic signals

- Conclusions
  - Demonstrated wave number estimates are an effective tool for tracking environmental range dependence
  - Perturbative inversion using qualitative regularization and approximate equality constraints can be used to produce accurate estimates of the seabed and water column sound speed profiles.